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PREFERENCE FOR WEALTH AND LIFE CYCLE PORTFOLIO CHOICE

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Preference for Wealth and Life Cycle Portfolio Choice

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Abstract

Do participation and investment in risky assets increase with wealth? Do the wealthiest households save at higher rates than the median households and is wealth more concentrated than earnings? Based on survey data, this paper shows that this is the case. Moreover, the paper provides a theoretical framework based on an extended version of the life-cycle model of consumption and portfolio choice that enables to explain differences in behavior between the wealthiest and others.

Keywords: Life-cycle, Portfolio Choice, Preference over Wealth, Wealth Inequality

JEL classification: D15, E21, G11 .

1 Introduction

Survey data on household behavior show that the investment in risky assets increases with wealth. Importantly, not only the stock market participation tends to be lower among poorer households, in addition, the portfolio share invested in risky assets rises in wealth among stockholders. Moreover, the wealthiest households save at higher rates than the median household (see e.g. Carroll, 2000; Dynan, Skinner, and Zeldes, 2004) and wealth is more concentrated than earnings (Diaz et al., 1997). The standard life-cycle model of consumption and portfolio choice has difficulties in explaining such differences in behavior between the wealthiest and others.

In this paper we investigate whether accounting for direct preferences over wealth provides insights in understanding the observed relation between portfolio choice and wealth as well as the observed concentration of wealth. In particular, we develop the informal model in Carroll (2000) into a fully-fledged quantitative life cycle model. This model assumes that wealth directly provides a stream of utility to consumers and treats it as a luxury good. The model implies that risk aversion is declining with wealth, since households are assumed to be less averse to risks over the luxury good (wealth) than over normal goods (standard consumption) and the former tends to dominate the period flow of utility as it becomes larger. At the same time, this assumption also implies that savings rates increase with wealth. This assumption alone, then, has the potential to jointly explain the high concentration of wealth at the top of the wealth distribution and the fact that the share of wealth invested in risky assets increases with wealth. While each of these insights in isolation is not new, we contribute to the literature by calibrating a quantitative life-cycle model and checking if there exist parametrizations of preferences over consumption and wealth such that it is possible to jointly match the main features of wealth concentration in the data and the pattern of portfolio shares over wealth levels for the top percentiles of the distribution. Except for the assumption on preferences, the model presented here is in most other respects standard. Agents receive a flow of earnings consisting of a deterministic hump-shaped profile and a stochastic component modeled by a permanent plus transitory shock, as is common in the literature (see, e.g. Cocco, Gomes and Maenhout, 2005, or Gomes and Michaelides, 2005).

In order to generate a realistic concentration of earnings, we add to this a fixed-level effect that is specific to the individuals and constant over the working life. There are two assets in the economy, a risk free and a risky assets. Markets are incomplete and trading in the assets is subject to non-negativity constraints. We focus only on stockholders, hence we do not assume any fixed participation cost and use as a reference population the population of stockholders only.

We simulate the model and search for a parametrization that allows it to match as well as possible both the wealth profile of the share of risky assets and the concentration of wealth at the top of the distribution. The result of this search is partially positive. Parametrizations exist that allow the model to match both statistics in the data up to the top 5 percent of the wealth distribution. Beyond that, while it is still possible to match the share of wealth of the highest percentiles of the distribution, it becomes impossible to have still increasing risky portfolio shares, suggesting that other mechanisms must be at play.

The second implication of the model augmented with direct preferences over wealth, is that it is able to explain also the higher concentration of wealth compared to income. This can be explained also by the standard model, although gaps are left for the share of the top percentiles of the wealth distribution.

Our paper aims at contributing to two different strands of literature, the first one is the household finance literature that has focused on life-cycle asset allocation between a risky and risk-free asset with labor income risk, the second one is the literature that has studied the forces behind the high concentration of wealth at the top of the distribution.

We contribute to the household finance literature that has extensively studied life-cycle asset allocation with labor income risk (e.g., Bodie, Merton and Samuelson 1992; Bertaut and Haliassos 1997; Viceira, 2001; Cocco Gomes and Maenhout, 2005). However, various implications of the standard model are at odds with the empirical evidence. Here, we focus on the fact that the portfolio share invested in risky assets is increasing with wealth conditional on participation. In this respect, our study is related to the recent papers by Benzoni, Collin-Dufresne and Goldstein (2007), Gomes and Michaelides (2003), Lynch and Tan (2011), Polkovnichenko (2007) Campanale, Fugazza and Gomes (2015) and Wacther and Yogo(2010) which have considered various explanations of observed patterns of risky investment over the life-cycle and over wealth levels. Benzoni et al.(2007) and Lynch and Tan(2011) consider alternative specifications of the labor income process, which can also deliver portfolio shares that are increasing in wealth, conditional on age. In Benzoni et al.(2007), this effect is driven by the low-frequency correlation between stock return and labor income, hence it only takes place early in life, since as the agent approaches retirement this correlation becomes irrelevant. In Lynch and Tan (2011) the result is driven by business cycle fluctuations in the conditional distribution of income shocks, and therefore the effect is again only present for young households. Gomes and Michaelides (2003) and Polkovnichenko (2007) generate a positive relation between the risky portfolio share and wealth by assuming habit formation preferences; however they point out that, in order to get strong effects within this model, the importance of the habit must be very high, and therefore it implies counter-factually high levels of wealth accumulation. Campanale, Fugazza and Gomes (2015) also obtain portfolio shares that are increasing in wealth by modeling different degrees of liquidity between risky and non-risky assets. Wachter and Yogo (2010) achieve the same result assuming multiple goods, and their model generates a positive relationship between wealth and the portfolio share of risky assets conditional on age.

In addition, we contribute to the literature that focuses on wealth concentration and savings behavior of the richest household. Carroll (2000) considers a model in which the extreme right skewness in the wealth distribution is generated by a mechanism that generates different savings behavior between high and low permanent income earners. De Nardi (2004) shows that, in an overlapping generations model of general equilibrium, voluntary bequests can explain the observed lifetime savings behavior and wealth concentration. Benhabib, Bisin and Luo (2019) show that stochastic earnings, differential savings, and capital income risk drive wealth dynamics in the United States and are consistent with the observed cross-sectional distribution of wealth and with the observed social mobility. In this paper, we show that direct preferences for wealth in a model with risky assets provide a relevant mechanism in explaining most of the concentration of wealth through both higher savings rates and higher returns to wealth from higher investment in risky assets.

The rest of the paper is organized as follows. In Section 1, we report the empirical evidence on household portfolio choice. In Section 3, we present the life-cycle model with direct preference for wealth. In Section 4, we introduce the calibration of the model. In Section 5, we discuss the results. Section 6 compares the predictions of the model with the empirical evidence. Section 7 concludes.

2 Data on Household Portfolio Choice

We pool data from the independent cross-sectional surveys in the Survey of Consumer Finances (SCF). The SCF is conducted by the Federal Reserve Board every three years to provide detailed information on family balance sheets, pensions and income of U.S. households. The SCF is nationally representative of households in the United States and collects detailed information on their characteristics and investment decisions. As in Wacther and Yogo (2010), we focus on the sub-sample of stockholders (i.e., households with positive risky assets). The survey is based on a sample design that oversamples relatively high-wealth households. In the following analysis and in the remaining part of the paper, we use the survey's sample weights to obtain unbiased statistics and to handle missing responses. We consider waves from 1989 through 2016. We keep for analysis households whose head is aged between 26 and 75 at the time of interview. We exclude households with non-positive net worth, as well as those who do not hold risky-assets from the sample. We convert nominal values to real 2015 dollars using Consumer Price Index. In this section, we provide the definition for the various components of wealth that we consider in our analysis. Total wealth (or net worth) is the sum of financial and nonfinancial assets minus all debt. Financial assets include liquid financial accounts, certificates of deposit, directly held bonds and stocks, mutual funds, retirement (both individual and employer-sponsored thrift-type) accounts, the cash value of life insurance, equity interest in trusts, annuities, and managed investment accounts. Nonfinancial assets include the primary residence, investment in real estate, and business equity. Debt includes mortgage and home equity loans for primary residence and investment real estate, credit card balances, and other loans. Risky assets is defined as the sum of public equity, investment in nonresidential real estate, business equity, and risky bonds. The risky asset share is the ratio between risky assets and total wealth. For the analysis conducted in the paper, we focus on the subsample of stockholders (i.e., households with positive investment in risky assets) with positive net worth.

2.1 Empirical Evidence

In this section, we present the empirical evidence on households portfolio choice focusing on the relation between risky assets and wealth. In particular, we extend the analysis of Wacther and Yogo (2010) to 2016.[1] We estimate a censored regression model in which the outcome variable is the risky portfolio share, i.e. the share of net worth invested in risky assets.

[1] Wacther and Yogo (2010) use SCF data from 1989 to 2004.

Table 1: Relation between risky portfolio share and net worth for stockholders

Explanatory variable	Main specification	Cohort Effects	Alternative definition of stockholding
Log net worth	0.111***	0.112***	0.0811***
	(200.6)	(200.8)	(111.3)
Age:			
24-35	-0.0236*	-0.0927***	-0.168***
	(-1.704)	(-6.382)	(-9.395)
34-45	0.0132	-0.0340***	-0.0676***
	(1.166)	(-2.949)	(-4.572)
54-65	-0.0819***	-0.0333***	-0.154***
	(-7.377)	(-2.957)	(-10.58)
64-75	-0.349***	-0.257***	-0.513***
	(-26.92)	(-19.40)	(-29.84)
Log net worth x Age:			
24-35	0.00754***	0.00705***	0.0167***
	(6.625)	(6.037)	(11.26)
34-45	0.00754***	0.00314***	0.00808***
	(3.091)	(3.662)	(7.262)
54-65	0.00111	0.000373	0.00619***
	(1.428)	(0.473)	(6.012)
64-75	0.0140***	0.0132***	0.0249***
	(15.59)	(14.6)	(20.92)
Non married	-0.0545***	-0.0555***	-0.110***
	(-20.02)	(-20.32)	(-29.35)
Household size			
1	0.0807***	0.0864***	0.0988***
	(22.85)	(24.35)	(20.42)
2	0.0255***	0.0307***	0.0172***
	(11.700)	(14.000)	(5.73)
3	0.00985***	0.0111***	-0.00187
	(4.121)	(4.650)	(-0.569)
5	-0.0108***	-0.0114***	-0.0356***
	(-3.594)	(-3.795)	(-8.624)
6 or more	-0.0161***	-0.0178***	-0.109***
	(-3.871)	(-4.256)	(-18.80)
Observations	211,989	211,989	210,607

The table reports results from a censored regression model for the risky portfolio share. Explanatory variables in the main specification are log net worth, age group, log net worth interacted with age group, marital status, household size, and interview-year dummies (not reported here). The omitted categories are: households with four members, whose had is aged 46-55 and married. In the second specification we include cohort dummies instead of interview-year dummies. In the third specifications we consider the risky portfolio share defined as the share of financial wealth invested in public equity instead of the share of net worth invested in risky assets. The sample consists of households with positive risky assets i the 1989-2016 Survey of Consumer Finances. The table reports the marginal effects with t-statistics in parenthesis (*** p< 0.01, ** p< 0.05, * p< 0.1).

The explanatory variables are log net worth, age group, log net worth interacted with age group, marital status, household size, and interview-year dummies. The omitted category is households with four members, whose head is aged 46–55 and married. Results are reported in Table 1. In the main specification (see column 1), the coefficient on log net worth is

0.111, which means that the portfolio share invested in risky assets increases in net worth for households aged 46–55. In particular, for an average household aged 46–55, a 100% rise in net worth is associated with a 10.7- percentage-point increase in the portfolio share. The coefficients on the interaction between log net worth and age groups are almost zero, which means that the positive relation between the risky portfolio share and net worth does not vary significantly over the life-cycle.

In the second specification reported in column 2 of Table 1, we include cohort dummies instead of interview-year dummies to examine the robustness of our findings. The relation between the risky portfolio share and net worth is robust to controlling for birth cohort.

In this paper, we use the share of net worth invested in risky assets as our primary measure of the risky portfolio share. A narrower measure is the share of financial wealth invested in public equity. In column 3 of Table 1, we focus this latter as a latent variable to examine the robustness of our findings. We still find a statistically significant relation between the portfolio share and net worth.

3 The Model

We model an investor who maximizes the expected discounted utility over her entire life. We assume that she derives utility from consumption of normal goods as well as directly from wealth treated as a luxury good. The instantaneous utility function U takes the form:

$$U(C_t, A_{t+1}) = \frac{C_t^{1-\rho}}{1-\rho} + \frac{(A_{t+1}+\gamma)^{1-\alpha}}{1-\alpha}$$

where the available financial wealth (before the realization of financial returns) is defined as $A_{t+1} = (X_t - C_t)$, X_t is available cash-on-hand at the beginning of t and C_t is the consumption stream in t. Utility from consumption is a standard CRRA function, with coefficient of relative risk aversion ρ. The utility derived from wealth is a modified Stone-Geary function, where α represents the relative risk aversion with respect to wealth. To capture the nature of wealth as luxury good, we follow Carroll (2000) and assume that $\rho > \alpha$

which implies that the marginal utility from consumption declines faster (as consumption increases) than the marginal utility from wealth (as wealth increases). The modified Stone-Geary parameter, $\gamma > 0$, represents a threshold level above which wealth is treated as luxury good and implying that the marginal utility of wealth is strictly lower than the marginal utility of consumption. The Stone-Geary parameter induces heterogeneous demand for wealth. Poor agents below the threshold will decumulate all wealth before dying, while individuals who have accumulated wealth above the threshold desire to accumulate more.

The effective length of her life, which lasts at most T periods, is governed by age-dependent life expectancy. At each date t, the survival probability of being alive at date $t+1$ is p_t (with $p_{t_0-1} = 1$). The investor starts working at age t_0 and retires with certainty at age $t_0 + K$. Investor's i preferences at date t are described by a time-separable utility function of the form:

$$\frac{C_{t_0}^{1-\rho}}{1-\rho} + \frac{(A_{t_0+1} + \gamma)^{1-\alpha}}{1-\alpha} + E_{t_0}\left[\sum_{j=1}^{T} \beta^j \left(\prod_{k=-1}^{j-2} p_{t_0+k}\right)\left(p_{t_0+j-1}\left(\frac{C_{t_0+j}^{1-\rho}}{1-\rho} + \frac{(A_{t_0+j+1} + \gamma)^{1-\alpha}}{1-\alpha}\right)\right)\right]$$

where $\beta < 1$ is a utility discount factor. Following Cocco, Gomes and Maenhout (2005), we do not model labor supply decisions and therefore ignore the insurance property of flexible work effort allowing investors to compensate for bad financial returns with higher labor income, as in Gomes, Kotlikoff and Viceira (2008).

3.1 Labor and retirement income

Available resources to finance consumption over the agent's life cycle derive from accumulated financial wealth and from the stream of labor income. At each date t during *working life*, the exogenous labor income is represented by

$$\log Y_{it} = f\left(t|\theta_j\right) + v_{it} + \epsilon_{i,t} \qquad t_0 \leq t \leq t_0 + K \tag{1}$$

where, $f\left(t|\theta_i\right)$ denotes the deterministic trend component , which depends on age t and is common to all individuals who share the same level of earnings ability which is maintained constant over the working life, θ_j, with $j = 1, ..., 10$[2]. Thus, we solve the model conditioning to the fixed earnings ability θ_j, with $j = 1, ..., 10$. The stochastic component, v_{it}, represents the permanent component which follows a random-walk process:

$$v_{it} = v_{it-1} + \omega_{i,t} \tag{2}$$

where u_{it} is distributed as $N(0, \sigma_\omega^2)$ and independent over time. Finally, ε_{it} represents pure transitory shocks distributed as $N(0, \sigma_\varepsilon^2)$ and independent over time.

During *retirement*, income is certain and equal to a fixed proportion λ of the permanent component of income in the last working year:

$$\log Y_{it} = \log \lambda + f\left(t_{0+K} | \theta_j\right) \qquad t_0 + K < t \leq T \tag{3}$$

where the level of the replacement rate λ is meant to capture at least some of the features of Social Security systems.

3.2 Investment opportunities

We allow savings to be invested in a short-term riskless asset, yielding each period a constant gross real return R^f, and one risky asset, characterized as "stocks" yielding stochastic gross real returns R_t^s. The excess returns of stocks over the riskless asset follows

$$R_t^s - R^f = \mu^s + \nu_t^s \tag{4}$$

where μ^s is the expected stock premium and ν_t^s is a normally distributed innovation, with mean zero and variance σ_s^2. We do not allow for excess return predictability and other forms of changing investment opportunities over time, as in Michaelides and Zhang (2017).

At the beginning of each period, financial resources available for consumption and saving are

[2] As specified in the next section, we proxy the effect of earnings ability by considering the age profiles of the ten deciles of labor earnings observed in the Survey of Consumer Finances

given by the sum of accumulated financial wealth W_{it} and current labor income Y_{it}, that we call *cash on hand* $X_{it} = W_{it} + Y_{it}$. Given the chosen level of current consumption, C_{it}, next period cash on hand is given by:

$$X_{it+1} = (X_{it} - C_{it})R^P_{it} + Y_{it+1} \tag{5}$$

where R^P_{it} is the portfolio return:

$$R^P_{it} = \alpha^s_{it} R^s_t + (1 - \alpha^s_{it})\, R^f \tag{6}$$

with α^s_{it} and $(1 - \alpha^s_{it})$ denoting the shares of the investor's portfolio invested in stocks and in the riskless asset respectively. We do not allow for short sales and assume that the investor is liquidity constrained, so that the nominal amount invested in each of then two financial assets are $B_{it} \geq 0$, $S_{it} \geq 0$, respectively for the riskless asset and stocks, are non negative in each period. All simulation results presented below are derived under the assumption that the investor's asset menu is the same during working life and retirement.

3.3 Solving the life-cycle problem

In this standard intertemporal optimization framework, the investor maximizes the expected discounted utility over life time, by choosing the consumption and the portfolio rules given uncertain labor income and asset returns. Formally, the optimization problem is written as:

$$\max_{\{C_{it}\}_{t_0}^{T-1}, \{\alpha^s_{it}\}_{t_0}^{T-1}} \left(\frac{C_{t_0}^{1-\rho}}{1-\rho} + \frac{(A_{t_0+1} + \gamma)^{1-\alpha}}{1-\alpha} + E_{t_0} \left[\sum_{j=1}^{T} \beta^j \left(\prod_{k=-1}^{j-2} p_{t_0+k} \right) \left(p_{t_0+j-1} \frac{C_{t_0+j}^{1-\rho}}{1-\rho} + \right. \right. \right.$$
$$\left. \left. \left. + p_{t_0+j-1} \frac{(A_{t_0+j+1} + \gamma)^{1-\alpha}}{1-\alpha} \right) \right] \right) \tag{7}$$

$$s.t. \quad X_{it+1} = (X_{it} - C_{it}) \left(\alpha^s_{it} R^s_t + (1 - \alpha^s_{it})\, R^f \right) + Y_{it+1} \tag{8}$$

with the labor income and retirement processes specified above and the no-short-sales and borrowing constraints imposed. Given its intertemporal nature, the problem can be restated in a recursive form, rewriting the value of the optimization problem at the beginning of period t as a function of the maximized current utility and of the value of the problem at $t+1$ (Bellman equation):

$$V_{it}\left(X_{it,}P_{it},\theta_j\right)=\max_{\{C_{it}\}_{t_0}^{T-1},\{\alpha_{it}^s\}_{t_0}^{T-1}}\left(\frac{C_{it}^{1-\gamma}}{1-\gamma}+\frac{(A_{t+1}+\gamma)^{1-\alpha}}{1-\alpha}+\beta E_t p_t V_{it+1}(X_{it_{+1}},P_{it+1})\right) \quad (9)$$

$$s.t. X_{it+1}=(X_{it}-C_{it})R_{it}^{P}+Y_{it+1} \quad (10)$$

This problem has no closed form solution: hence, the optimal values for consumption and portfolio shares depending on the values of each state variable at each point in time are obtained by means of numerical techniques. To this aim, we apply the standard backward induction procedure starting form the last possible period of life T. The optimal consumption and portfolio share policy rules are obtained for each possible value of the continuous state variables (X_{it} and P_{it}) using the standard grid search method. Going backwards, for every period $t=T-1,T-2,...,t_0$, the Bellman equation (9) is used to obtain the optimal rules for consumption and portfolio shares.

4 Calibration

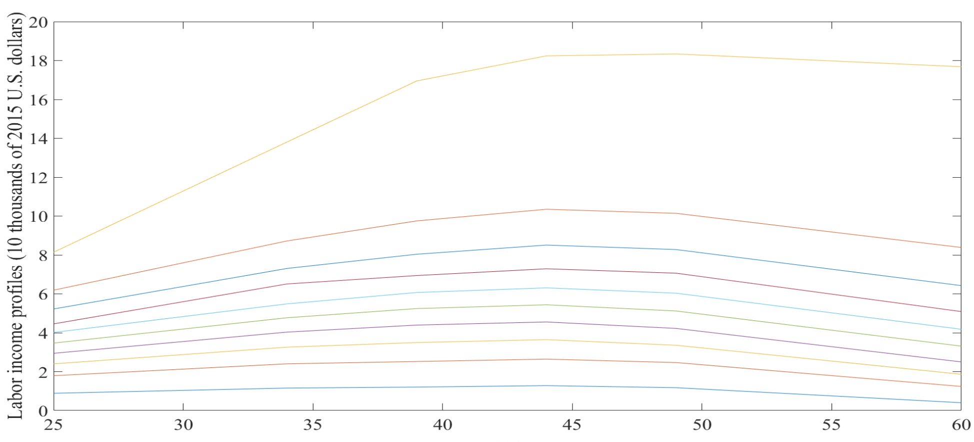
Parameter calibration concerns investor's preferences, the features of the labor income process during working life and retirement, and the moments of the risky asset returns.

The investor begins her working life at the age of 26 and works for (a maximum of) 35 periods (K) before retiring at the age of 60. After retirement, she can live for a maximum of 15 periods until the age of 75. In each period, we take the conditional probability of being alive in the next period p_t from the life expectancy tables of the US *National Center for Health*

Statistics. The preference parameters, the coefficient of relative risk aversion with respect to consumption and wealth, ρ and α, are set equal to 5 and 3 respectively, and the threshold level of wealth, γ, is set equal to 9 millions U.S. dollars. The subjective discount factor, β, is 0.96. The riskless (constant) interest rate is set at 0.02, with expected stock premium μ^s fixed at 0.04 . The standard deviation of the returns innovations is set at $\sigma_s = 0.157$. Finally, we impose a zero correlation between stock return innovations and aggregate permanent labor income disturbances ($\rho_{sY} = 0$).

The labor income process is calibrated as follows. The deterministic trend component, $f(t, \theta_i)$, is modelled for ten possible levels of earnings ability (θ) and is calibrated considering the deterministic life-cycle earnings profiles at different deciles, computed on the Survey and Consumer Finances data (waves from 1989 to 2016). We deflate nominal labor earnings to real 2015 dollars using the CPI index for all urban consumers. Thus, we solve the life-cycle consumption and portfolio choice problem for each decile assuming that agents stay in the same decile of the deterministic component of labor income for their whole lifetime. In figure 1, we report the labor income profiles.

Figure 1: Life-cycle labor income profiles by deciles



The parameters of the permanent and the transitory stochastic component are calibrated as in Cocco, Gomes and Maenhout (2005). After retirement, income is a constant proportion λ of the final (permanent) labor income, with $\lambda = 0.68$. Given this calibration, the simulated life-cycle earnings display the distribution reported in table 2 and reflect the earnings inequality observed in U.S. with the Gini index equal to 0.55.

Table 2: Distribution of earnings

Shares of earnings	**0-25**	**25-50**	**50-75**	**75-100**	**95-100**	**Gini**
Survey of Consumer Finances	0.05	0.12	0.20	0.62	0.29	0.55
All simulations	0.03	0.10	0.22	0.65	0.29	0.55

The table reports the shares of earnings by percentiles and the Gini index of earnings that result from the calibrated labor income process. For reference, the corresponding quantities observed in the 1989–2016 Survey of Consumer Finances are reported.

Table 3 reports all the calibrated parameters used to solve the model.

Table 3: Calibrated parameters

Description	Parameter	Value
Working life (max)	T	26 -60
Retirement (max)	$t_0 + K$	61 -75
Discount factor	β	0.96
Risk aversion with respect to consumption	ρ	5
Risk aversion with respect to wealth	α	3
Threshold level of wealth (ten thousands of 2015 U.S. dollars	γ	900
Replacement ratio	λ	0.68
Variance of permanent shocks to labor income	σ^2_ω	0.0106
Variance of transitory shocks to labor income	σ^2_ϵ	0.0738
Riskless rate	r	0.02
Excess returns on stocks	μ^s	0.04
Variance of stock returns innovations	σ^2_s	0.025

5 Results

We solve the life-cycle problem through standard numerical dynamic programming techniques, as described. This section describes the optimal policies for consumption and portfolio choice.

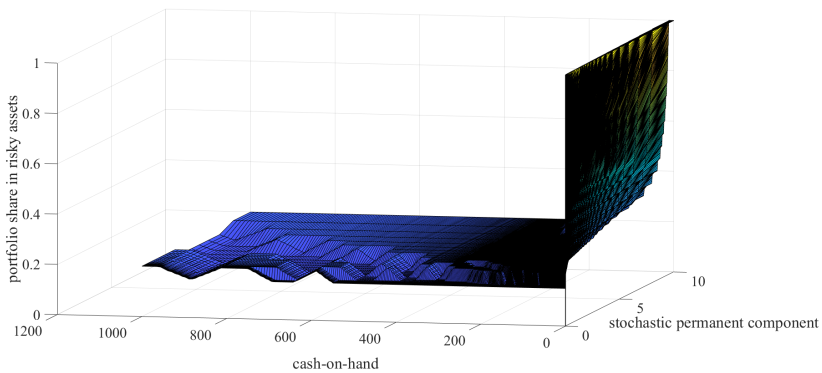
5.1 Decision Rules

In this section, we discuss the decision rules for the benchmark model and for the model augmented with direct utility from wealth reported in Figures 2 and 3, respectively. The risk aversion with respect to consumption, ρ, is set to 5 in both cases. For the model with direct preference for wealth, the risk aversion with respect to wealth, α, is 3 and the threshold level of wealth, γ, is set to 9 millions of US dollars. The decision rules are depicted for households at age 50.

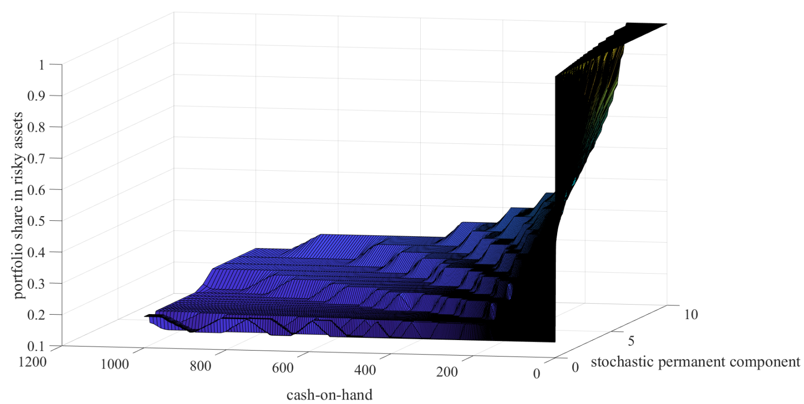
The decision rules for the standard life cycle model with no direct preference for wealth are reported in Figure 2. The optimal share invested in risky assets is plotted for the lowest decile of the deterministic component of labor income and for its median value (panel a) and b), respectively). The optimal risky portfolio share is monotonically declining in cash-on-hand. Given that human wealth is relatively less risky than stocks it acts as a substitute for bonds, thereby, the lower the cash-on-hand, the higher the implicit holding in bonds (as a share of total wealth), so the higher the optimal investment in stocks.

Figure 2: Optimal portfolio policy with no direct preference for wealth

(a) Deterministic labor income: lowest decile, θ_1



(b) Deterministic labor income: median, θ_5

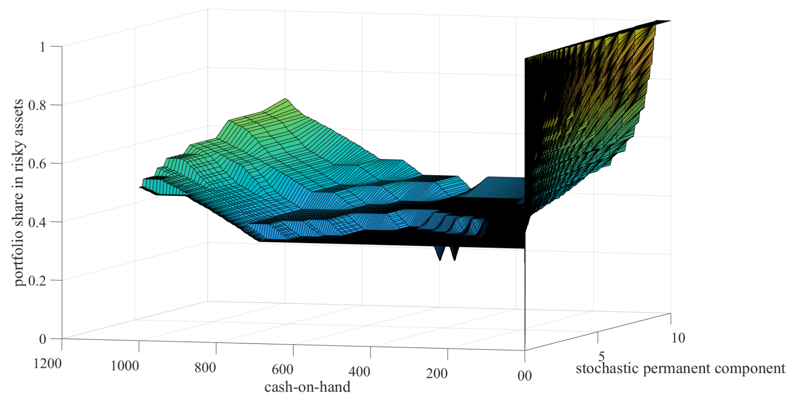


This figure shows the optimal policy for the risky portfolio share at age 50 for the life cycle model with no direct preference over wealth. The control variable is the optimal portfolio share invested in risky assets. The state variables are cash-on-hand and the level of the stochastic permanent component of labor income. The optimal share invested in risky assets is plotted for the lowest decile and for the median of the deterministic component of labor income, i.e., for the lowest level of earnings ability θ_1 and for the median level of earnings ability θ_5. The household receives stochastic labor income from age 26 through 65 and retirement income from age 66 through 76. The risk aversion, ρ is equal to 5 and the discount factor, β, is equal to 0.96. Cash-on-hand and labor income are expressed in ten thousand of 2015 U.S. dollars.

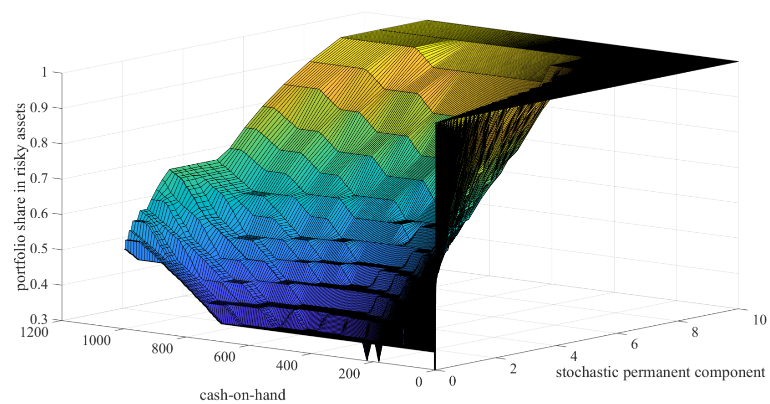
Compared to the well-known results from the standard life cycle model, the decision rules for the model with direct utility from wealth, are non monotonically decreasing in cash on hand (see Figure 3). The risky portfolio share is declining in wealth for relatively low and very high levels of cash on hand, while in the intermediate region it is increasing in wealth. For relatively poorer individuals, the risk aversion with respect to consumption (γ) prevails over their risk aversion with respect to wealth (α) and thus they soon reduce their investing in stocks as cash-on-hand rises as in the standard life cycle model. Over a certain threshold of cash-on-hand, the preference for wealth prevails over other reasons for saving (i.e. and they start to save at higher rates). Thus, at higher level of cash-on-hand the investment behavior is driven by the direct utility derived from wealth and by the relatively lower risk aversion coefficient (α) over wealth. Thus the preference for risk rises with wealth which induces to increase the optimal stock investing as cash-on-hand increases. For very high levels of cash-on-hand, the risky portfolio share is declining in cash-on-hand being entirely determined by the wealth-related component of the utility function.

Figure 3: Optimal portfolio policy with direct preference for wealth

(a) Deterministic labor income: lowest decile, θ_1



(b) Deterministic labor income: median, θ_5



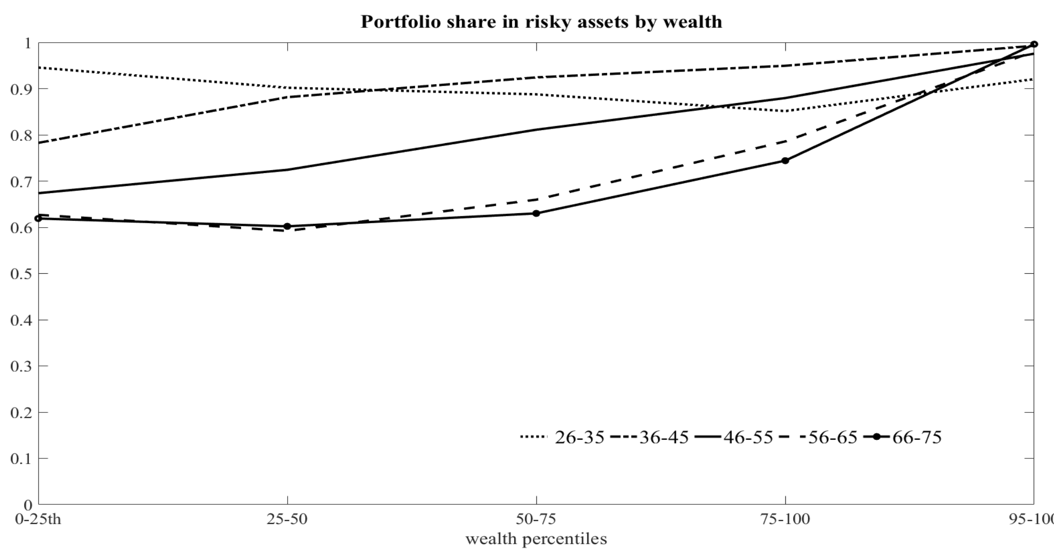
This figure shows the optimal policy for the risky portfolio share at age 50 for the life cycle model with preferences over wealth. The control variable is the optimal portfolio share invested in risky assets. The state variables are cash-on-hand and the level of the stochastic permanent component of labor income. The optimal share invested in risky assets is plotted for the lowest decile and for the median of the deterministic component of labor income, i.e., for the lowest level of earnings ability θ_1 and for the median level of earnings ability θ_5. The household receives stochastic labor income from age 26 through 65 and retirement income from age 66 through 76. The risk aversion, ρ is equal to 5 and the discount factor, β, is equal to 0.96. Cash-on-hand and labor income are expressed in ten thousand of 2015 U.S. dollars.

5.2 Simulation results

In this section, we discuss the relationship between risky portfolio shares and wealth implied by simulating the calibrated model above.

Figure 4 shows the relation between the optimal portfolio share invested in risky assets and wealth quartiles and the top 5 wealth percentiles, for five age groups, from 26 to 75. Our results confirm that direct wealth preference is one possible explanation for the increasing relation between risky investment and wealth. Not surprisingly, the optimal behavior at young ages appears not to be affected by the presence of direct utility from accumulated wealth and the two profiles are almost indistinguishable. At young ages, the low levels of accumulated wealth induces investors to behave as ordinary investors, even in the case in which they derive utility directly from wealth. As investors grow older the optimal stock investing declines for the standard human capital explanation. At mature ages, when accumulated wealth has overcome the threshold, investors who derive utility from wealth save at a higher rate which fast leads them into the region where decision rules are increasing in wealth. During retirement, the preference for wealth prevents investors from decumulating as in the standard model which translates into higher levels of wealth held at all ages and in a relatively lower fraction of accumulated wealth invested in risky assets.

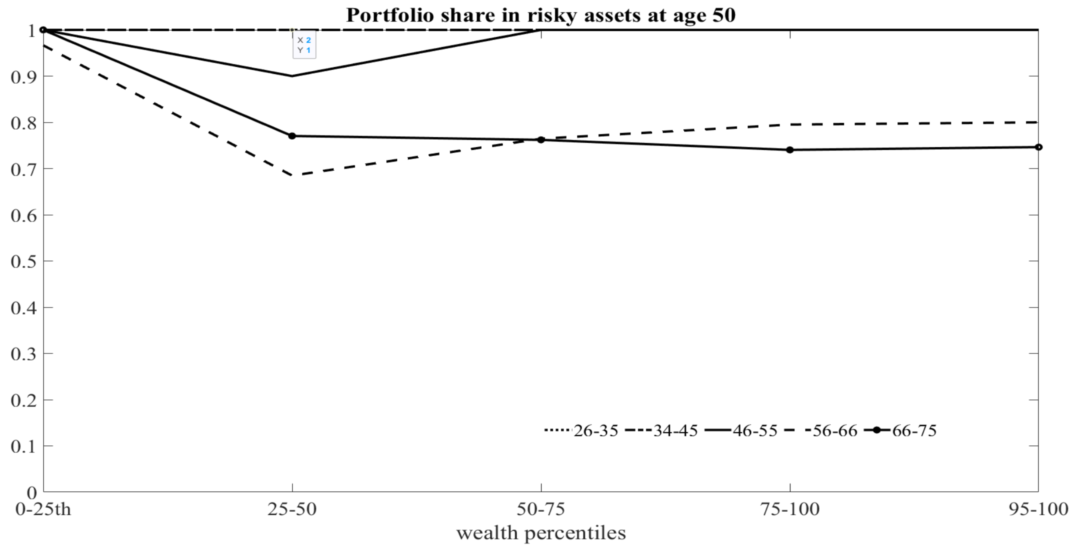
Figure 4: Risky portfolio share and wealth in the model with direct preference for wealth



This figure shows the relation between the optimal portfolio share invested in risky assets and wealth quartiles and the top 5 wealth percentiles. Five age groups, from 26 to 75, are considered.

For comparison, figure 5, reports the relation between the risky portfolio share and wealth implied by the model with no direct preference for wealth. The standard life-cycle model implies that risky portfolio share is almost flat in wealth.

Figure 5: Risky portfolio share and wealth in the model with no direct preference for wealth



This figure shows the relation between the optimal portfolio share invested in risky assets and wealth quartiles and the top 5 wealth percentiles. Five age groups, from 26 to 75, are considered.

5.2.1 Wealth accumulation and inequality

The model with a direct preference for wealth is able to match the wealth inequality (measured by the Gini index), observed in the data for stockholders in the US. [3]. However, the preference for wealth implies a wealth accumulation slightly higher than what observed in the data. The wealth-to-income ratio for stockholders observed in US in 2016 is about 7.5, while the value implied by our calibration is about 9.4 (see Table 4)[4].

[3] To make comparison with data, we focus on the subsample of stockholders, i.e. households with positive risky assets defined as the sum of public equity, investment in nonresidential real estate, business equity, and risky bonds.

[4] As discussed in section 2, wealth is total net worth and income is total labor earnings – included earnings from private business.

Table 4: Wealth accumulation and inequality in the life-cycle model

	Shares of wealth					Gini	Wealth-to-income ratio
	0-25	**25-50**	**50-75**	**75-100**	**95-100**		
Survey of Consumer Finances	0.01	0.05	0.11	0.83	0.49	0.80	7.50
Life-cycle model with direct preference for wealth	0.01	0.06	0.14	0.78	0.52	0.79	9.40
Life-cycle model with no direct preference for wealth	0.02	0.08	0.21	0.69	0.29	0.60	3.10

The table reports the distribution of wealth, the Gini coefficient for wealth and the wealth-to-income ratio in the Survey of Consumer Finances (2016) and for the simulated life-cycle models. In the case with direct preference for wealth, risk aversion on consumption is $\rho = 5$, risk aversion on wealth is $\alpha = 3$, the threshold level of wealth is $\gamma = 900$ and the discount factor $\beta = 0.96$. In the case of non-direct preference for wealth, risk aversion on consumption is $\rho = 5$ and the discount factor, β is 0.96.

6 Matching the empirical evidence

In this section we consider the calibration of the model with a direct preference for wealth that better matches the relation between risky portfolio share investment wealth as well as the wealth accumulation and inequality observed in the data. We then compare the predictions of this calibrated model to the data and to the predictions of the standard life-cycle model with no direct preference for wealth. Table 5 reports the values of parameters that allow the better match as well as the target moments for the wealth distribution.

Table 5: Wealth accumulation and inequality in the life-cycle model

	Shares of wealth					Gini	Wealth to income ratio
	0-25	**25-50**	**50-75**	**75-100**	**95-100**		
Survey of Consumer Finances	0.01	0.05	0.11	0.83	0.49	0.80	7.50
Life cycle model with direct preference for wealth							
$\rho = 14; = 6; \gamma = 900; \beta = 0.75$	0.01	0.05	0.12	0.82	0.64	0.79	9.90
Life-cycle model with no direct preference for wealth							
$\gamma = 14; \beta = 0.75$	0.03	0.10	0.23	0.64	0.27	0.54	7.30

The table reports the distribution of wealth, the Gini coefficient for earnings and wealth and the wealth-to-income ratio for stockholders in the Survey of Consumer Finances (2016) and for the simulated life cycle models. In the model with direct preference for wealth, risk aversion on consumption is $\rho = 14$, risk aversion on wealth is $\alpha = 6.5$, the threshold level of wealth is $\gamma = 900$ and $\beta = 0.75$. In the model with no-direct preference for wealth, risk aversion on consumption is $\rho = 14$ and the discount factor is $\beta = 0.75$. Stockholders are households with positive risky assets defined as the sum of public equity, investment in nonresidential real estate, business equity, and risky bonds.

We now focus on the relation between risky portfolio share and wealth. In Panel A of Table 6, we report the median portfolio share invested in risky assets observed for stockholders in the Survey of Consumer Finances from 1989 to 2016. The medians are tabulated by age group and wealth quartile. Stockholders are households with positive risky assets defined as the sum of public equity, investment in nonresidential real estate, business equity, and risky bonds. For the age group 36–45, the portfolio share invested in risky assets is 28% for the lowest wealth quartile and 43% for the highest quartile. The portfolio share is 64% for the top fifth percentile of net worth. For the age group 56–65, the risky portfolio share is 20% for the lowest wealth quartile and 47% for the highest quartile. The risky portfolio share is 66% for the top fifth percentile of net worth. In Panel B, we report the portfolio share implied by the model with a direct preference for wealth. The risky portfolio share is increasing in wealth for all but the youngest age group, which is substantially consistent with the empirical evidence shown in Panel A. For the age group 36–45, the risk portfolio share is 34% for the lowest wealth quartile and 37% for the highest quartile. The risky portfolio share is 75% for the top fifth percentile of wealth. For the age group 56–65, the risky portfolio share is 23% for the lowest wealth quartile and 30% for the highest quartile. The risky portfolio share is 57% for the top fifth percentile of wealth. The relationship between wealth and median risky portfolio shares obtained from the life-cycle model with direct preference for wealth are also depicted in figure 6. From our discussion in section 5, the risk attitude varies with wealth. For poor investors who behave as ordinary savers and accumulate for precautionary and retirement purposes, the relative risk aversion over consumption prevails in determining their attitude towards risky assets. For wealthier investors who display an additional motive to save, the relatively lower risk aversion over wealth induce them to increase the investment in risky asset. For younger investors the first effect dominates, implying the standard results of risky portfolio share declining in wealth. As households grow older and permanent income shocks accumulate, the second effect dominates leading to a positive relation between risky portfolio share and wealth. For comparison, in Panel C, we report the risky portfolio share implied by the standard life-cycle model with no direct preference for wealth. In this case, the risky portfolio share is flat or moderately rising for older investors.

Figure 6: Risky portfolio share and wealth

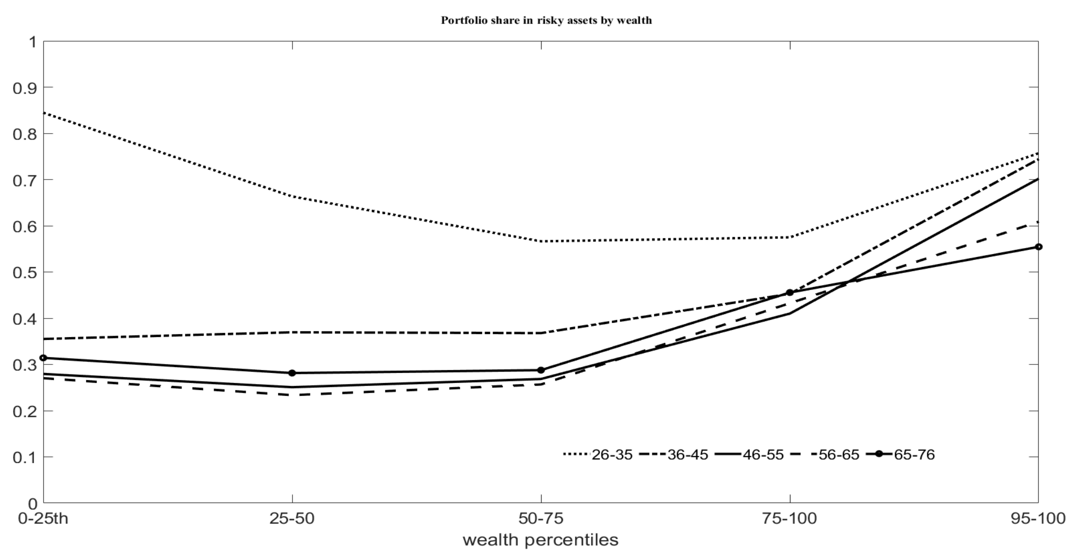


Table 6: Risky portfolio share in the life-cycle model

	Age				
Percentile of net worth	26-35	36-45	46-55	56-65	66-75
Panel A: Survey of Consumer Finances (stockholders only)					
0-25	0.29	0.28	0.23	0.20	0.15
25-50	0.21	0.20	0.21	0.20	0.17
50-75	0.24	0.26	0.29	0.29	0.25
75-100	0.35	0.43	0.47	0.47	0.43
top 5	0.64	0.66	0.68	0.66	0.63
All	0.26	0.29	0.30	0.29	0.25
Panel B: Life cycle model with direct preference for wealth					
0-25	1.00	0.34	0.25	0.23	0.29
25-50	0.66	0.35	0.25	0.22	0.25
50-75	0.55	0.35	0.25	0.25	0.26
75-100	0.52	0.37	0.30	0.30	0.35
top 5	0.78	0.75	0.64	0.57	0.55
All	0.63	0.35	0.25	0.25	0.28
Panel C: Life cycle model with no direct preference for wealth					
0-25	1.00	0.30	0.22	0.18	0.20
25-50	0.76	0.31	0.23	0.20	0.20
50-75	0.56	0.30	0.24	0.20	0.22
75-100	0.46	0.30	0.24	0.20	0.20
top 5	0.45	0.30	0.25	0.23	0.24
All	0.60	0.30	0.23	0.20	0.20

The table reports median risky portfolio shares (i.e., risky assets as a percentage of net worth). In Panel A, we report the median portfolio share in the 1989–2016 Survey of Consumer Finances for stockholders sorted into age groups (columns), then into quartiles of net worth within each age group. Stockholders are households with positive risky assets defined as the sum of public equity, investment in nonresidential real estate, business equity, and risky bonds. Panel B and Panel C report median risky portfolio shares for households simulated in the life-cycle model with no-direct preference for wealth and with a direct preference for wealth, respectively.

7 Conclusions

In this paper, we show that a direct, additively separable, preference for wealth is able to provide an explanation for the observed cross-sectional variation in portfolio behavior. Our model provides a mechanism for high income households to continue accumulating assets even when they have reached their buffer stock targets and saved sufficiently for retirement. In this way, the model with a direct preference for wealth can explain several features of the data that the standard life-cycle model fails to explain. In particular, it can explain most of the concentration of wealth and increasing risk tolerance with wealth, which leads to a positive relation between risky share investment and wealth.

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