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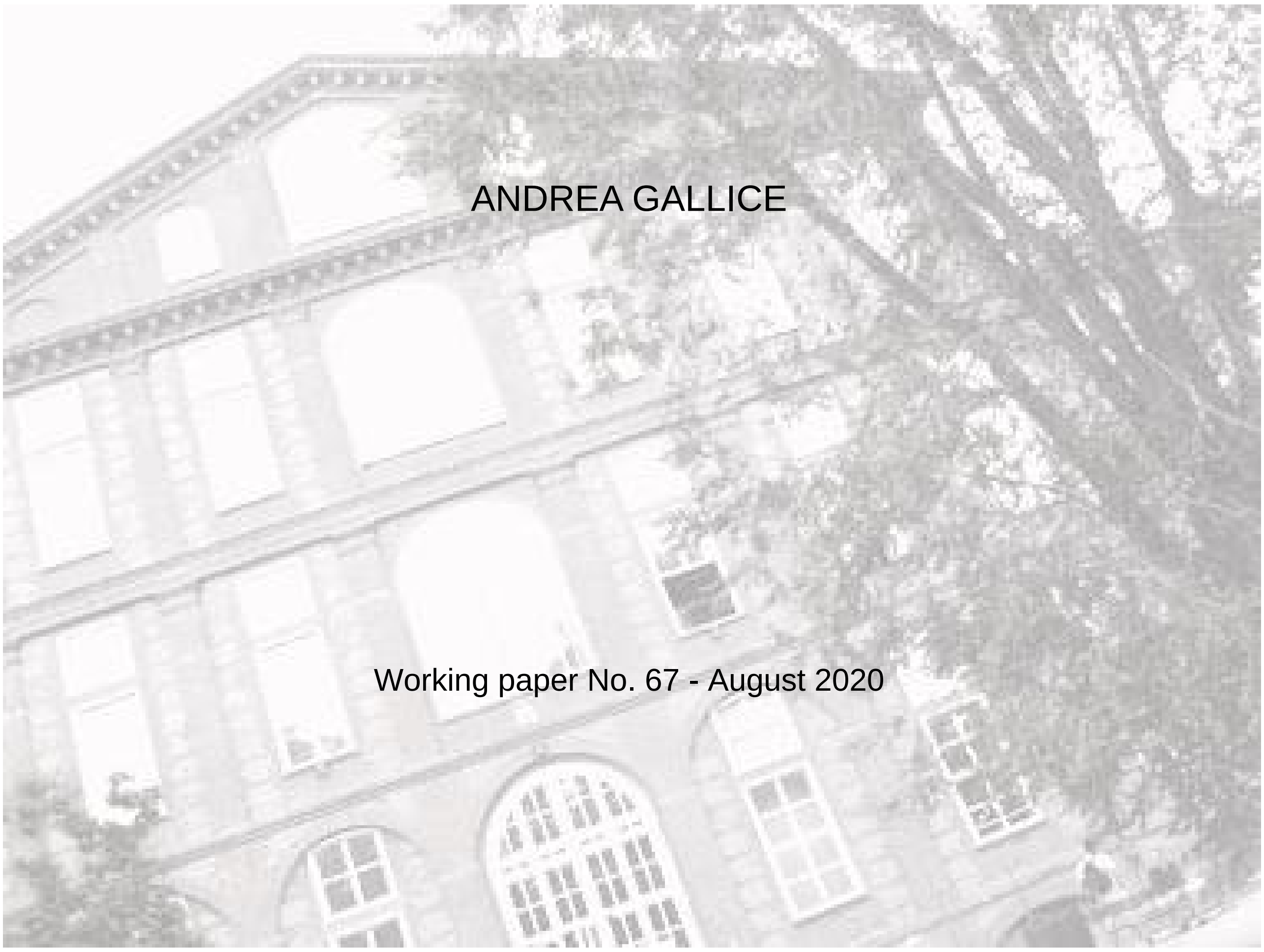
SELF-SERVING BIASED REFERENCE POINTS IN BANKRUPTCY PROBLEMS



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Self-Serving Biased Reference Points in Bankruptcy Problems*

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Abstract

I formalize the notion of self-serving bias within the framework of reference-dependent preferences. Self-serving bias affects agents' expectations in a systematic way and, through this channel, it influences their reference points. I then apply the model to bankruptcy problems and provide a ranking of standard allocative rules on the basis of the level of welfare that they generate.

Keywords: self-serving bias, reference dependent preferences, bankruptcy problems.

JEL classification: D03, D63.

1 Introduction

Self-serving bias (SSB) is a pervasive phenomenon that influences individual behavior in a variety of ways: people tend to overestimate their own merits and abilities, to favorably acquire and interpret information, to give biased judgments about what is fair and what is not, and to inflate their claims and contributions.[1] As such, SSB can have important social and economic implications. For instance, it is considered

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[1] Research in psychology and sociology provides many convincing examples of the existence of such a bias. For instance, Svenson (1981) reports that the overwhelming majority of subjects (93%) feel they drive better than average, while Ross and Sicoly (1979) show how, for married couples, the sum of the their self-assessed personal contributions to household tasks usually exceeds 100%.

one of the main causes of costly impasses in bargaining and negotiation (Babcock *et al.*, 1995; Babcock and Loewenstein, 1997; Farmer *et al.*, 2004), and it can be a source of political instability (Passarelli and Tabellini, 2013). Moreover, it has been argued that SSB increases the propensity to strike (Babcock *et al.*, 1996), the incidence of trials (Farmer and Pecorino, 2002), and the intensity of marital conflicts (Schütz, 1999).

While the importance of SSB is widely acknowledged in the literature, a proper formalization of the concept, and an analytical study of its implications, continue to be scarce and case-specific. I introduce a general framework that combines SSB with the notion of reference-dependent preferences. More precisely, I postulate that SSB affects the way agents set their reference points in a simple but systematic way. The bias influences agents' expectations and, through this channel, ultimately determines their reference points. A self-serving biased agent has biased expectations, i.e., expectations that foresee a more favorable outcome than a rational assessment of the situation would warrant. As such, the agent unconsciously sets an inflated reference point. I investigate some implications of the model, both at the individual and the aggregate level.

I then apply the model to a bankruptcy problem, a typical situation where the hypothesis that agents may have self-serving biased reference points seems particularly appropriate.[2] In a bankruptcy problem an arbitrator must allocate a scarce resource among a finite number of claimants. I study how the arbitrator's problem gets modified when claimants have biased reference points and provide a clear ranking of standard allocative rules in terms of the level of utilitarian welfare that they generate. More precisely, I show that when all the claimants are biased in the same proportional way, the constrained equal losses rule dominates the proportional rule, which in turn dominates the constrained equal awards rule.

[2] Gallice (2011) also considers an application to litigations in the courtroom. Gallice (2012) instead investigates the *strategic* reasons (as opposed to the unconscious role of the self-serving bias) that can push litigants to inflate their claims.

2 Reference-Dependent Preferences and Self-Serving Bias

The paper introduces the notion of self-serving bias (SSB) within a framework of reference-dependent preferences (RDPs). As a model of the latter, I follow the fruitful approach introduced by Koszegi and Rabin (2006), which I briefly review in Section 2.1. I then discuss the issue of reference point formation in Section 2.2. Finally, in Section 2.3, I define the notion of self-serving biased reference points.

2.1 A model of reference-dependent preferences

Koszegi and Rabin (2006) (KR06 in what follows) introduce a formal model that captures the notion of RDPs: an individual's perception of a given outcome is shaped not only by the outcome per se but also by how this outcome compares with some reference point.

More precisely, the KR06 model postulates a utility function $u(x \mid r)$ (to be shortly defined), where $x \in \mathbb{R}^K$ is a K-dimensional outcome and $r \in \mathbb{R}^K$ is the reference point. The model allows for stochastic outcomes (such as lotteries defined over deterministic outcomes) as well as for stochastic reference points. If x is distributed according to F and r is distributed according to G, then the agent's expected utility is given by:

$$U(F|G) = \int \int u(x \mid r) dG(r) dF(x)$$

where the function $u(x \mid r)$ takes the following form:

$$u(x \mid r) = \sum\nolimits_k m_k(x_k) + \sum\nolimits_k \mu(m_k(x_k) - m_k(r_k))$$

The strictly increasing function $m_k(\cdot)$ captures the direct effect that the possession or consumption of good x_k has on $u(x \mid r)$. The function $\mu(\cdot)$ is a "universal gain-loss function". Given the reference point r_k, $\mu(\cdot)$ reflects the additional effects that perceived gains or losses have on $u(\cdot)$. More precisely, and in line with the original prospect theory formulation of Kahneman and Tversky (1979), $\mu(\cdot)$ is assumed to satisfy the following properties:

P1: $\mu(z)$ is continuous for all z, strictly increasing and such that $\mu(0) = 0$.

P2: $\mu(z)$ is twice differentiable for $z \neq 0$.

P3: $\mu''(z) > 0$ if $z < 0$ and $\mu''(z) < 0$ if $z > 0$.

P4: if $y > z > 0$ then $\mu(y) + \mu(-y) < \mu(z) + \mu(-z)$.

P5: $\lim_{z\to 0^-} \mu'(z) / \lim_{z\to 0^+} \mu'(z) \equiv \lambda > 1$.

The $\mu(\cdot)$ function thus displays a kink when $z = 0$, i.e., when the actual outcome x_k matches the reference point r_k. Property P3 then indicates that $\mu(\cdot)$ is convex for values of x_k that are below r_k (domain of losses) and concave for values of x_k that are above r_k (domain of gains). The same property also implies that the marginal influence of these perceived gains and losses is decreasing. Property P4 means that, for large absolute values of z, the function $\mu(\cdot)$ is more sensitive to losses than to gains. P5 implies the same result for small values of z: $\mu(\cdot)$ is steeper approaching the reference point from the left (losses) than from the right (gains). Taken together, these last two properties capture the loss aversion phenomenon.

2.2 Reference points as (rational) expectations

A key aspect of any model of RDPs is specifying how an agent comes to define his reference point. Kahneman and Tversky (1979) proposed the so-called status quo formulation, which states that individuals set their reference points in line with what they are used to. An alternative possibility is that agents define their reference points according to what they expect rather than to what they have. Consider, for instance, the situation of a worker who expects a wage increase of \$500 but then actually gets an increase of just \$200; this outcome is likely to feel more like a loss with respect to expectations rather than a gain with respect to the status quo.

Supported by recent empirical evidence (Abeler *et al.*, 2011; Ericson and Fuster, 2011), the general consensus now acknowledges the role of expectations as the main determinant of reference points. However, theoretical models that try to embed such a feature face an additional challenge. In order to get sensible results and falsifiable predictions, these models must analytically define agents' reference points. Therefore, they need to identify the precise nature of an agent's expectations. The natural approach, then, is to rely on the notion of rational expectations. For instance, the KR06 model postulates that the reference point is defined by the rational expectations the agent held in the recent past about the outcome of the problem at issue. As such, KR06 endogenize the reference point: in their notion of personal equilibrium, an agent expects to implement those actions (and thus reach those outcomes) that he indeed finds optimal to pursue in the specific "state of the world" that will occur. While KR06 considers nonstrategic situations with only one individual, Shalev (2000) investigates the issue of reference points formation when the final outcome is determined by the interaction of different agents. He also defines reference points through rational expectations. In particular, he introduces the concept of loss-aversion equilibrium, i.e., an equilibrium in which an agent's reference point coincides with the actual outcome of a modified game where the original payoff is adjusted to capture the loss that the agent expects to experience.

In this paper, I also define reference points as expectations. However, agents' expectations (and thus their reference points) will not necessarily be rational, precisely because they may be unconsciously influenced by SSB.[3]

2.3 Self-serving biased reference points

I first formalize the concept of rational reference point. I then use this notion as a benchmark to define the alternative concept of self-serving biased reference point.

Consider a set of agents $N = \{1, ..., n\}$ whose preferences are defined over the possible allocations of an endowment of size $E \neq 0$. Let

[3] Ahmad (2020) follows a similar approach as he investigates the role of self-serving biased reference points in $k-$double auctions. He endogenizes the "true"reference point by letting it coincide with the expected price of the good in equilibrium and then lets self-serving bias exogenously distorts this estimate in the agent's favor.

$$X = \left\{ (x_1, ..., x_n) \mid x_i \cdot E \geq 0 \text{ for all } i \in N \text{ and } \sum_{i=1}^{n} x_i = E \right\}$$

denote the set of feasible and non-wasteful allocations, where x_i is the amount received by agent $i \in N$.[4] The actual allocation is determined by a draw from a probability distribution θ which is defined on X, i.e., $\theta(x_1, ..., x_n)$ is the probability that allocation $(x_1, ..., x_n)$ emerges and $\int_X \theta(x_1, ..., x_n)\, dx = 1$. We define an agent's rational reference point as his expected allocation given θ.

Definition 1 *A rational agent has a reference point* $r_i^{rat} = \int_X x_i \theta(x_1, ..., x_n)\, dx$.

Having defined the concept of rational reference point, I then argue that SSB affects agents' reference points in a simple but systematic way. In line with Babcock and Loewenstein's (1997, p. 110) definition of SSB as a tendency "to conflate what is fair with what benefits oneself", I claim that, everything else being equal, a self-serving biased agent has a higher reference point with respect to the one that his hypothetical unbiased alter ego would set through rational expectations.

Definition 2 *A self-serving biased agent has a reference point* $r_i^{ssb} > r_i^{rat}$.

Clearly, the definition does not analytically pin down a unique value for a self-serving biased reference point. Indeed, any reference point $r_i > r_i^{rat}$ qualifies as a biased reference point.[5] Obviously, the larger the difference $\Delta r_i = \left(r_i^{ssb} - r_i^{rat}\right)$, the larger the agent's bias.

Definition 2 implies that, in the context of RDPs, SSB negatively affects an agent's utility and this negative effect is increasing in the size of the agent's bias. The bias in fact inflates the reference point and thus leads to either smaller perceived

[4] In the definition of X, the feasibility condition $x_i \cdot E \geq 0$ for all $i \in N$ requires that the amount x_i that generic agent i gets must be non-negative whenever E is positive, while on the contrary x_i must be non-positive whenever E is negative.

[5] For instance, our definition of biased reference point is consistent with the approach that Hart and Moore (2008) pursue in their study about the pros and cons of flexible contracts. The authors assume in fact that parties set as their reference point the *best* outcome that the contract permits, which is then obviously larger than the average outcome.

gains or larger perceived losses. Proposition 1 formalizes this result (all proofs appear in the appendix).

Proposition 1 *Let agent i have RDPs such that $u_i(x_i \mid r_i) = m(x_i) + \mu(m(x_i) - m(r_i))$. Then, $u_i(x_i \mid r_i^{ssb}) < u_i(x_i \mid r_i^{rat})$ for any possible x_i. Moreover, the agent's utility is strictly decreasing in $\Delta r_i = \left(r_i^{ssb} - r_i^{rat}\right)$.*

Definition 2 also illustrates a more general result: whenever all agents are rational then their reference points are mutually compatible, i.e., their sum matches the size of the resource to be shared. This simple consideration leads to the statement of the following proposition:

Proposition 2 *Consider a profile of reference points $r = (r_1, ..., r_n)$ and let $E \neq 0$ be the size of the available surplus. Then, if $\sum_i r_i = E$, all agents have rational reference points. If instead $\sum_i r_i > E$, at least some of the agents have self-serving biased reference points.*

In what follows I apply the framework of self-serving biased reference points to the analysis of bankruptcy problems.

3 An Application to Bankruptcy Problems

In a bankruptcy problem an arbitrator must allocate an endowment of size E among a number of agents whose claims sum up to more than E. A typical example is a bankrupt firm that must be liquidated. The proposed framework of self-serving biased reference points seems particularly appropriate in the context of bankruptcy problems. These are, in fact, typical situations in which the two conditions that underlie the model are likely to hold. First, a claimant's utility is likely to be affected not only by the actual allocation but also by how this compares with his expectations (i.e., his reference point).[6] Second, a claimant's expectations are likely to be affected

[6] Gallice (2019) provides a fully-fledged analysis of bankruptcy problems with reference-dependent preferences. He considers as possible specifications for claimants' reference points the claims vector, the zero awards vector, the minimal rights vector, and claimants' beliefs about the awards vector that the arbitrator will implement.

by SSB such that the agent may have inflated reference points. For instance, the creditor of a bankrupt firm may think that his claims deserve a higher priority compared to those of other claimants and may thus expect a larger reimbursement.

I model the problem as follows: an arbitrator must allocate a homogeneous and perfectly divisible endowment whose size I normalize to $E = 1$ among $n \geq 2$ claimants. Let $c = (c_1, ..., c_n)$ with $c_i \in \mathbb{R}_+$ and such that $\sum_i c_i > 1$ be a vector that collects individual claims. The vector $x = (x_1, ..., x_n)$ with $\sum_i x_i = 1$ denotes instead a possible allocation. Claimants have RDPs, i.e., $u_i(x_i \mid r_i) = m(x_i) + \mu(m(x_i) - m(r_i))$ where, as before, r_i is the agent's reference point. In what follows I actually set $m(x_i) = x_i$ such that $u_i(x_i \mid r_i) = x_i + \mu(x_i - r_i)$.[7]

I am then interested in studying some welfare properties of three standard allocative rules that are commonly advocated in the literature (see Thomson, 2003 and 2015 for exhaustive reviews). These rules are:

- The *proportional* rule (*prop*), which allocates amounts proportional to claims:

$$x_i^{prop} = \lambda^{prop} c_i \text{ with } \sum\nolimits_i \lambda^{prop} c_i = 1$$

- The *constrained equal awards* rule (*cea*), which assigns equal amounts to all claimants subject to no one receiving more than his claim:

$$x_i^{cea} = \min\{c_i, \lambda^{cea}\} \text{ with } \sum\nolimits_i \min\{c_i, \lambda^{cea}\} = 1$$

- The *constrained equal losses* rule (*cel*), which assigns equal amount of losses to all claimants subject to no one receiving a negative amount:

$$x_i^{cel} = \max\left\{0, c_i - \lambda^{cel}\right\} \text{ with } \sum\nolimits_i \max\left\{0, c_i - \lambda^{cel}\right\} = 1$$

All three rules select allocations that satisfy some basic desirable properties

[7] This assumption simplifies the analysis as the linear form of $m(\cdot)$ implies that the utility function $u_i(x_i \mid r_i)$ satisfies the same properties that characterize the $\mu(\cdot)$ function (see Proposition 2 in KR06). However, to measure a claimant's standard utility in terms of the amount of resource that he obtains is consistent with the approach that is usually adopted in bankruptcy problems (see Thomson, 2015).

(Thomson, 2015), such as *non-negativity* (no agent is asked to pay: $x_i^\tau \geq 0$ for any $i \in N$ and any $\tau \in \{prop, cea, cel\}$), *claims boundedness* (no agent receives more than his claim: $x_i^\tau \leq c_i$ for any i and any τ), and *balance* (the arbitrator allocates all the resource: $\sum_i x_i^\tau = 1$ for any τ).[8]

As a measure of welfare, I use the utilitarian social welfare function whose generic form is given by $W_{ut}(x) = \sum_i u_i(x_i)$. Therefore, and given the balance property, $W_{ut}(x^\tau) = 1 + \sum_i \mu(x_i^\tau - r_i)$ for any $\tau \in \{prop, cea, cel\}$.

3.1 The case with rational claimants

Rational claimants have rational reference points. Let $\theta(x^\tau) \in [0,1]$ denote the probability that the arbitrator will implement allocation x^τ with $\tau \in \{prop, cea, cel\}$ (or, equivalently, claimants' beliefs that the arbitrator is of type τ). In line with Definition 1, a claimant's rational reference point is then given by $r_i^{rat} = \sum_\tau x_i^\tau \theta(x^\tau)$ with $\sum_i r_i^{rat} = 1$.[9] Given that the rational reference point of agent i is a weighted average of all the possible realizations of x_i and since $x_i^\tau \leq c_i$ for any τ (claim boundedness property), I can conclude that $r_i^{rat} \leq c_i$ for any i. In other words, a rational claimant realizes that he will possibly get less than his claim. As such, he sets his expectations, and thus his reference point, accordingly.

When all the agents are rational two other interesting relations hold. First, the actual gains or losses that an agent will perceive conditional on the specific allocation that the arbitrator will implement cancel out across rules. More formally, $\sum_\tau \left(x_i^\tau - r_i^{rat}\right) = 0$ for any $i \in N$. This implies that in expectations an agent does not experience any gain or loss. Second, within any rule, individual gains and losses also cancel out across agents, i.e., $\sum_i \left(x_i^\tau - r_i^{rat}\right) = 0$ for any τ. However, this last condition does not necessarily imply the condition $\sum_i \mu(x_i^\tau - r_i^{rat}) = 0$ given that the μ function weights gains and losses differently. It follows, that in

[8] As an example of how the three rules work in practice, consider a bankruptcy problem with $n = 3$ and let $c = (0.3, 0.5, 0.8)$. The rules would then select the following allocations: $x^{prop} = (0.1875, 0.3125, 0.5)$, $x^{cea} = (0.3, 0.35, 0.35)$, and $x^{cel} = (0.1, 0.3, 0.6)$.

[9] Following on the example that was introduced in the previous footnote, assume $\theta(x^\tau) = \frac{1}{3}$ for any $\tau \in \{prop, cea, cel\}$. The vector of rational reference points is then given by $r^{rat} = (0.196, 0.321, 0.483)$. Clearly, $\sum_i r_i^{rat} = 1$.

general $W_{ut}(x^\tau) \neq 1$.[10] Since losses loom larger than gains, one may actually expect $W_{ut}(x^\tau) < 1$ most of the times.[11]

3.2 The case with self-serving biased claimants

I now study the bankruptcy problem when some of the claimants have a self-serving biased reference point. The first result that I show is quite straightforward: the bias is welfare detrimental not only at the individual level (see Proposition 1) but also at the aggregate one.

Proposition 3 *$W_{ut}(x^\tau \mid r') < W_{ut}(x^\tau \mid r^{rat})$ for any rule $\tau \in \{prop, cea, cel\}$ and any vector $r' \geq r^{rat}$ with $r' \neq r^{rat}$.*

It is then interesting to study how the different rules perform in terms of the actual level of welfare that they generate. I tackle this issue in a restricted domain where it is possible to define a clear ranking. In particular, I focus on the case in which claimants know with certainty the principal's type, i.e., $\theta(x^\tau) = 1$ for some $\tau \in \{prop, cea, cel\}$. This implies that, from the point of view of the claimants, the outcome of the allocation procedure is deterministic such that $r_i^{rat} = x_i^\tau$. I then further assume that all claimants are biased in the same proportional way. More formally, $r_i^{ssb} = (1+\beta) r_i^{rat}$ for all $i \in N$ where the parameter $\beta > 0$ captures the proportion by which an agent's reference point is inflated with respect to the rational benchmark.

Within such a frawework, utilitarian welfare can be expressed as $W_{ut}(x^\tau) = 1 + \sum_i \mu(x_i^\tau - r_i^{ssb}) = 1 + \sum_i \mu(-\beta x_i^\tau)$. The distribution of perceived losses across

[10] Indeed, $W_{ut}(x^\tau) = 1$ only in some specific situations such as when $\theta(x^\tau) = 1$ for some $\tau \in \{prop, cea, cel\}$ (i.e., no uncertainty about the arbitrator's type) or when $c_i = c_j$ for all $i, j \in N$ (i.e., claimants are symmetric such that $x^\tau = \left(\frac{1}{n}, ..., \frac{1}{n}\right)$ for any τ and thus $r^{rat} = x^\tau$).

[11] For instance, it is always the case that $W_{ut}(x^\tau) < 1$ when $n = 2$ and $c_i \neq c_j$ as the perceived loss of agent i weights more than the perceived gain of agent j. On the contrary, the condition $W_{ut}(x^\tau) > 1$ verifies only in specific situations. A necessary condition is that the number of agents that experience a gain is larger than the number of those that experience a loss. The intuition is that, due to diminishing sensitivity of the μ function (Property P3), the aggregate benefit experienced by a large number of claimants that receive little more than their reference point may overcome the negative effect experienced by a small number of agents that suffer large losses.

agents (i.e., the distribution of $-\beta x_i^\tau$) is thus proportional to the actual distribution that rule τ implements (i.e., the distribution of x_i^τ). Because of the diminishing sensitivity to losses displayed by the μ function, such a relationship leads to a clear ranking of the three allocative rules. The first step toward this goal is to evaluate how evenly the different rules allocate the contested resource across claimants. As a measure of inequality I use the standard Lorenz dominance criterion (see Sen, 1973), which I briefly review.

Consider two allocations $x^\tau = (x_1^\tau, ..., x_n^\tau)$ and $x^\kappa = (x_1^\kappa, ..., x_n^\kappa)$ with $\tau, \kappa \in \{prop, cea, cel\}$ and then define the new vectors $\hat{x}^\tau = (\hat{x}_1^\tau, ..., \hat{x}_n^\tau)$ and $\hat{x}^\kappa = (\hat{x}_1^\kappa, ..., \hat{x}_n^\kappa)$ which display the coordinates of the original vectors in increasing order, i.e., $\hat{x}_i^\tau \leq \hat{x}_{i+1}^\tau$ and $\hat{x}_i^\kappa \leq \hat{x}_{i+1}^\kappa$ for any $i \in \{1, ..., n-1\}$. I say that x^τ Lorenz dominates x^κ, and I denote such a relation by writing $x^\tau \succ_L x^\kappa$, if:

$$\hat{x}_1^\tau \geq \hat{x}_1^\kappa, \quad \hat{x}_1^\tau + \hat{x}_2^\tau \geq \hat{x}_1^\kappa + \hat{x}_2^\kappa, \quad ... \quad , \text{ and } \hat{x}_1^\tau + ... + \hat{x}_{n-1}^\tau \geq \hat{x}_1^\kappa + ... + \hat{x}_{n-1}^\kappa$$

with at least one strict inequality. I instead write $x^\tau \succeq_L x^\kappa$ if none of the above mentioned inequalities holds strictly. I can then say that rule τ Lorenz dominates rule κ, a relation that I denote as $\tau \succ_L \kappa$, whenever $x^\tau \succeq_L x^\kappa$ for any possible vector of claims and $x^\tau \succ_L x^\kappa$ for at least one vector of claims. In other words, the relation $\tau \succ_L \kappa$ indicates that rule τ always implements an allocation that is (weakly) less skewed with respect to the allocation that rule κ implements. Bosmans and Lauwers (2011) show that the constrained equal awards rule Lorenz dominates the proportional rule, which in turn Lorenz dominates the constrained equal losses rule. More formally, $cea \succ_L prop \succ_L cel$. Such a result plays a key role in the proof of the following proposition:

Proposition 4 *When all claimants have a self-serving biased reference point* $r_i^{ssb} = (1+\beta)\, r_i^{rat}$ *with* $\beta > 0$ *and* $\theta(x^\tau) = 1$ *for some* $\tau \in \{prop, cea, cel\}$*, the following ranking emerges:* $W_{ut}(x^{cel}) \geq W_{ut}(x^{prop}) \geq W_{ut}(x^{cea})$.

This result highlights a peculiar characteristic of bankruptcy problems when claimants have RDPs and biased reference points. From a purely utilitarian point of view, the allocations that generate the highest level of welfare are those that implement the most uneven distributions. Because of the diminishing sensitivity of the gain-loss function $\mu(\cdot)$ (Property P3), it is in fact more efficient to disappoint a lot just a few claimants rather than disappoint a little all of them.

4 Conclusions

I introduced the notion of self-serving bias within the framework of reference-dependent preferences by arguing that the bias systematically inflates agents' expectations, and thus their reference points. I applied the model to the analysis of bankruptcy problems and studied some welfare properties of standard allocative rules in this enriched setting. Self-serving biased reference points are likely to play a role also in other contexts such as disputes and litigations, auctions, and bargaining problems. The proposed framework can thus help to analytically investigate the consequences of this pervasive bias also in these situations.

Appendix

Proof of Proposition 1

The $m(\cdot)$ function is strictly increasing. The $\mu(\cdot)$ function is also strictly increasing. Therefore, $\mu(\cdot)$ is strictly decreasing in r_i. By Definition 2, $r_i^{ssb} > r_i^{rat}$. It follows that, for any given x_i, $\mu(m(x_i) - m(r_i^{ssb})) < \mu(m(x_i) - m(r_i^{rat}))$ which implies $u_i(x_i \mid r_i^{ssb}) < u_i(x_i \mid r_i^{rat})$. More in general, the fact that $\mu(\cdot)$ is strictly decreasing in r_i implies that $\frac{\partial u_i(x_i|r_i^{ssb})}{\partial \Delta r_i} < 0$ for any x_i.

Proof of Proposition 2

By Definition 1, $r_i^{rat} = \int_X x_i \theta\left(x_1, ..., x_n\right) dx$. Therefore,

$$\begin{aligned}\sum\nolimits_i r_i^{rat} = \sum\nolimits_i \left(\int_X x_i \theta\left(x_1, ..., x_n\right) dx\right) = \\ = \int_X \left(\sum\nolimits_i x_i\right) \theta\left(x_1, ..., x_n\right) dx = \\ = \int_X E \cdot \theta\left(x_1, ..., x_n\right) dx = E\end{aligned}$$

By Definition 2, a self-serving biased agent has instead a reference point $r_i^{ssb} > r_i^{rat}$. It follows that $\sum_i r_i > E$ whenever there is at least one agent $i \in N$ such that $r_i = r_i^{ssb}$.

Proof of Proposition 3

Let $W_{ut}\left(x^\tau \mid r^{rat}\right) = 1 + \sum_i \mu(x_i^\tau - r_i^{rat})$ be the level of welfare generated by rule $\tau \in \{prop, cea, cel\}$ when all the claimants have rational reference points. Now consider the vector $r' \geq r^{rat}$ such that there exists at least one claimant whose reference point is biased, i.e., $r_i' = r_i^{ssb} > r_i^{rat}$ for some $i \in N$. Let $W_{ut}(x^\tau \mid r') = 1 + \sum_i \mu(x_i^\tau - r_i')$ be the associated level of welfare. Notice that $\sum_i \left(x_i^\tau - r_i^{rat}\right) = 0$ while $\sum_i \left(x_i^\tau - r_i'\right) < 0$ given that $\sum_i r_i' > 1$ (see Proposition 2). Therefore, it must be the case that $\left(x_i^\tau - r_i'\right) < \left(x_i^\tau - r_i^{rat}\right)$ for some $i \in N$, which in turn implies $\mu\left(x_i^\tau - r_i'\right) < \mu\left(x_i^\tau - r_i^{rat}\right)$ for some $i \in N$ given that $\mu(\cdot)$ is strictly increasing (Property P1). It follows that $W_{ut}(x^\tau \mid r') < W_{ut}(x^\tau \mid r^{rat})$.

Proof of Proposition 4

Given that $r_i^{ssb} = (1+\beta)\, r_i^{rat}$ for all $i \in N$ and $\theta\left(x^\tau\right) = 1$ for some $\tau \in \{prop, cea, cel\}$, utilitarian welfare is given by $W_{ut}\left(x^\tau\right) = 1 + \sum_i \mu(x_i^\tau - r_i^{ssb}) = 1 + \sum_i \mu(-\beta x_i^\tau)$ where $\mu(-\beta x_i^\tau) < 0$ such that $\sum_i \mu(-\beta x_i^\tau) < 0$. $W_{ut}\left(x^\tau\right)$ is strictly increasing in $\sum_i \mu(-\beta x_i^\tau)$. Because of the strict convexity of the μ function in the domain of losses (Property P3), we have that $\mu(a)+\mu(b) < \mu(a-\epsilon)+\mu(b+\epsilon) < 0$ for any $a \leq b < 0$ and $\epsilon \in (0, b)$. Therefore, the rule that achieves the highest $\sum_i \mu(-\beta x_i^\tau)$ is the rule that implements the more skewed distribution of $-\beta x_i^\tau$. Given that $cea \succ_L prop \succ_L cel$

(Bosmans and Lauwers, 2011) and $\beta > 0$, it follows that $-\beta x^{cea} \succ_L -\beta x^{prop} \succ_L -\beta x^{cel}$. Therefore, $\sum_i \mu(-\beta x_i^{cea}) \leq \sum_i \mu(-\beta x_i^{prop}) \leq \sum_i \mu(-\beta x_i^{cel}) < 0$ such that $W_{ut}(x^{cel}) \geq W_{ut}(x^{prop}) \geq W_{ut}(x^{cea})$.

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