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LIFE-CYCLE WEALTH ACCUMULATION AND CONSUMPTION INSURANCE

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Life-cycle Wealth Accumulation and Consumption Insurance

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Abstract

Households appear to smooth consumption in the face of income shocks much more than implied by life-cycle versions of the standard incomplete market model under reference calibrations. In the current paper we explore in detail the role played by the life-cycle profile of wealth accumulation. We show that a standard model parameterized to match the latter can rationalize between 83 and more than 97 percent of the consumption insurance against permanent earnings shocks empirically estimated by Blundell, Pistaferri and Preston (2008), depending on the tightness of the borrowing limit.

Keywords: precautionary savings, Epstein-Zin, consumption insurance coefficients, life-cycle
JEL Codes: E21

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All errors are our own.

1 Introduction

Economic theory suggests that agents endowed with concave utility functions try to smooth out fluctuations in income. The extent to which consumption ends up being smoother than income depends on both the concavity of the utility function and the kind of insurance instruments that are available, both informal or formal through financial markets. The life-cycle version of the standard incomplete market model — henceforth SIM model —, currently the workhorse of heterogeneous agents macroeconomics, when parameterized according to reference values used in the literature falls significantly short of the empirical values of insurance against permanent earnings shocks, as estimated by Blundell *et al.* (2008). In this kind of model wealth, in the form of a single asset, and debt are used to smooth consumption in the face of earnings fluctuations. In the current research we revisit the SIM model by focussing specifically on the role that life-cycle wealth accumulation plays in determining the degree of consumption smoothing. To preview the results, we find that when the model is calibrated so that it matches the whole empirical profile of wealth accumulation over the working life, a version of the model with the tightest borrowing constraint can match up to 83 percent of the Blundell *et al.* (2008) — henceforth BPP — estimates of insurance against permanent earnings shocks. When the model is solved under the loosest borrowing limit it can virtually match the empirical values.

In order to study the role of the life-cycle pattern of wealth accumulation we modify the baseline self-insurance model by moving from standard expected utility to Epstein-Zin preferences. The economies studied in this research are otherwise standard. They feature a large number of ex-ante identical agents. Agents have finite lives and go through the two stages of life of working age and retirement. During working life they receive an exogenous stochastic stream of earnings that cannot be insured due to incomplete markets. During retirement they receive a constant pension benefit that depends on the full history of the household's earnings. They have access to a single risk-free asset that they can use to smooth consumption in the face of variable earnings, subject to a borrowing constraint. The model is cast in a partial equilibrium framework.

The key feature of Epstein-Zin preferences is that contrary to standard expected utility preferences, they permit a complete separation between the elasticity of inter-temporal substitution (EIS) and risk-aversion. In the context of the present model, this allows us to keep the EIS at a relatively high value while raising risk aversion as much as is needed to explain the empirically observed insurance coefficients without an excessive accumulation of wealth. As a result, for example, a calibration with an EIS of 0.8 allows us to match the insurance coefficient for permanent shocks of 0.36 with a risk aversion of 20 while still keeping the discount factor above 0.9 and matching the empirical average wealth-to-income ratio. By contrast, a comparable model with expected utility and risk aversion of 20 would require a discount factor of 0.545 to match a realistic wealth-to-income ratio and would still fall short of the insurance coefficient estimated in the data. The intuition behind this result is that in the Epstein-Zin case, raising risk aversion while keeping the elasticity of inter-temporal substitution high, allows the model to increase early life precautionary savings without concurrently creating a strong motive for holding large retirement wealth. This allows the model to match the empirical wealth-to-income ratio with plausible values for patience. At the same time, the combination of high risk aversion, high patience and the willingness to accept inter-temporal redistribution of consumption away from young ages reshuffles wealth towards the early part of the life-cycle when it is most needed for insurance purposes and away from middle age, when the accumulation of retirement wealth and the lower effective residual persistence of the shocks makes the latter more easily insurable. For this reason, this mechanism increases the insurance coefficients for permanent shocks in the first part of the life-cycle without affecting those in mid-life. This has the effect of raising the average coefficients and at the same time of making their age profile flatter, hence closer to the flat profile found in the data.

Given that the main mechanism that allows the model to generate insurance coefficients that are in line with the data is the redistribution of wealth across different periods of the life-cycle, it is important to verify that the resulting life-cycle profiles of wealth match the data. For this reason we next solve a preferred calibration of the model where the coefficients of risk aversion and the inter-temporal elasticity of substitution are chosen so as to minimize the distance between the model and data wealth accumulation profiles during working life. In this case we find that 83 percent of the insurance coefficients against permanent earnings shocks measured by BPP can be rationalized by the model with a zero borrowing limit. This figure raises to over 97 percent in the version of the model where borrowing is allowed subject only to the constraint that the household is able to repay for sure. In light of this latter result we can say that one key finding of the present work is that the failure of the baseline model to match the empirical insurance coefficients for permanent shock is due to its inability to generate the right amount of wealth accumulation early in the life cycle.

Measuring and studying insurance coefficients has proven challenging for two reasons. First, high quality panel data on both consumption and earnings are needed and, second, the problem of identifying different shocks from the observable income process must be circumvented. The first problem arises because the two main data sets used to study household behavior in the US, that is the Panel Study on Income Dynamics (PSID) and the Consumer Expenditure Survey (CEX) respectively lack consumption data or the panel dimension. With respect to the first issue, an example of an early effort in this sense is Attanasio and Davis (1996) who used several issues of the CEX to construct synthetic cohorts and study how the evolution of between groups earnings inequality translated into consumption inequality. Strictly speaking though, this does not measure insurance of shocks per se. With respect to the second issue, efforts have been made to distinguish between permanent and temporary shocks by using proxies like disability and short unemployment spells respectively (Dynarski *et al.*, 1997). Alternatively, others like Krueger and Perri (2006) have chosen to simply analyze the response of consumption to income shocks without trying to identify the different shocks.

A major step forward was made by Blundell *et al.* (2008), that used the CEX to estimate a food demand equation and then applied its inverse to PSID data on food consumption, thus obtaining an artificial data set with both a panel dimension and joint data on consumption and income. This, coupled with a suitable strategy to identify shocks, allowed them to come up with a first estimate of insurance coefficients. Kaplan and Violante (2010) first evaluated the standard SIM model against the data to test if it can match BPP estimates of the insurance coefficients. They found that under standard parameterizations this model can explain between 19 and 61 percent of the empirical estimates of insurance coefficients against permanent shocks, depending on the assumption of the zero or natural borrowing constraint respectively. In the wake of their paper, a few other quantitative papers have been written to extend the basic SIM model to better fit insurance data. Among those, we can cite Cerletti and Pijoan-Mas (2012) who extended the model to explore the role of non-durable goods and the adjustment in the consumption bundle that this allows and Karahan and Ozkan (2013) who estimated an earnings process featuring age-varying persistence and showed that this improves the life-cycle profiles of insurance coefficients of an otherwise standard model.

Finally, more recently a parallel line of research that uses wages rather than earnings as primitives and studies the extent of insurance against wage shocks has developed. In this line of research Blundell *et al.* (2016) provided benchmark empirical estimates and Wu and Krueger (2018) developed a first quantitative study to test an extension of the SIM model, featuring households with double earners, against those estimates.

The present paper is most closely related to Kaplan and Violante (2010) in that it also constructs a quantitative SIM model to study its implications for the insurance coefficients using earnings as the primitive shocks. There are three main differences between our study and theirs. First, we assume Epstein-Zin preferences instead of standard expected utility. Second, while Kaplan and Violante (2010) offers a broad theoretic-quantitative analysis of the problem, our research is more focused on trying to explain the gap between model and data estimates of insurance coefficients of permanent shocks, that has proven harder to bridge.[1] Third and key to pursuing that goal, we try to match the whole model pattern of wealth accumulation over the working life, rather than simply constraining the average wealth-to-income ratio.

The rest of the paper is organized in the following way. Section 2 is devoted to explaining the model, section 3 presents the calibration and section 4 discusses the results. Finally, in section 5 a brief conclusion is outlined.

2 Model

We consider a standard life-cycle economy with uninsurable idiosyncratic earnings risk and borrowing constraints. We assume that agents are endowed with Epstein-Zin preferences. In each period agents choose the optimal allocation of their resources between consumption expenditures and savings. There is a single asset that can be used for saving in the economy. The model is cast in partial equilibrium and there is no aggregate uncertainty. A cohort of agents is simulated and the model-generated patterns of consumption insurance are studied.

[1] An alternative view has been put forth by Hryshko and Manovskii (2017) who claim that the observed gap is an artifact of averaging PSID estimates across the groups of what are termed "sample" and "non-sample" households, characterized by very different earnings processes and behavior.

2.1 Demographics and preferences

Time is discrete with model periods of one year length. The model is populated by a continuum of households. Agents live for a maximum of $T = 80$ model periods. They enter the model at age 20 and retire after $T^{ret} = 45$ years of work. In each period of life t they face a probability π_{t+1} of surviving one more year. Agents care only about their own consumption and do not value leisure, hence they supply inelastically their unitary endowment of time.

Households value the uncertain stream of future consumption according to the following inter-temporal utility function:

$$V_t(S_t) = \{c_t^\gamma + \beta E[\pi_{t+1} V_{t+1}^\alpha(S_{t+1})]^{\frac{\gamma}{\alpha}}\}^{\frac{1}{\gamma}} \tag{1}$$

where the variable S_t represents the set of all past histories of shocks up to age t and initial assets that can at each age be summarized into three state variables. As it will become clear in the next few sections, these state variables are cash-on-hand at the beginning of the period, the value of the permanent earnings shock and the average past realizations of gross labor earnings. In the above representation of utility γ is the parameter that controls the elasticity of substitution between current consumption and the certainty equivalent of future utility, the elasticity of substitution being given by $\frac{1}{1-\gamma}$. On the other hand, α is the parameter that controls the curvature of the future utility certainty equivalent function and corresponds to a risk aversion of $1 - \alpha$. Finally, the parameter β determines the weight of future versus current utility and represents the subjective discount factor. In the expression above the expectation E is taken with respect to histories S_{t+1} up to $t + 1$ conditional on history S_t being realized up to age t.

2.2 Income process

During working life agents receive a stochastic flow of net earnings Y_{it} which can be expressed as:

$$logY_{it} = g_t + y_{it} \tag{2}$$

and

$$y_{it} = z_{it} + \varepsilon_{it} \tag{3}$$

where g_t is a deterministic component common to all households and y_{it} is the stochastic component of the labor income. In turn, the stochastic component can be decomposed into a transitory part ε_{it} and a permanent part z_{it} that follows the process:

$$z_{it} = z_{i,t-1} + \zeta_{it} \tag{4}$$

The initial realization of the permanent component is drawn from an initial distribution with mean 0 and variance σ^2_{z0}. The shocks ε_{it} and ζ_{it} are normally distributed with mean 0 and variances σ^2_ε and σ^2_ζ, are independent of each other, over time and across agents. Retired households receive a fixed pension benefit $P(\overrightarrow{Y}_i)$ where $\overrightarrow{Y}_i$ is the vector collecting all the realizations of gross earnings for agent i, that is, the pension benefit is a function of the history of all past earnings. Agents can save in a single asset. We denote the amount of the asset held by household i at age t with A_{it} and assume that the asset pays a constant return r. We assume that a borrowing constraint $A_{it} \geq \underline{A}$ holds. The household's budget constraint can then be written:

$$C_{it} + A_{i,t+1} = (1+r)A_{it} + I_{it}Y_{it} + (1 - I_{it})P(\overrightarrow{Y}_i) \tag{5}$$

where I_{it} is an indicator function that takes a value of 1 if $T < T^{ret}$ and 0 otherwise.

2.3 Household's optimization problem

With the description of the model given above and omitting for simplicity of notation the index i for the household, we can write the optimization problem at each age. This will be described by the Bellman equation:

$$V_t(X_t, z_t, \overline{Y}_t) = \max_{c_t, A_{t+1}} \{c_t^\gamma + \beta E[\pi_{t+1} V^\alpha_{t+1}(X_{t+1}, z_{t+1}, \overline{Y}_{t+1})]^{\frac{\gamma}{\alpha}}\}^{\frac{1}{\gamma}} \tag{6}$$

where V_t is the value function at age t and the state variables are current cash-on-hand X_t, the realization of the permanent component of the earnings process z_t, and the average of past gross earnings realizations up to age t denoted with $\overline{Y}_t$. The households maximize the CES aggregator of current consumption and the certainty equivalent of future utility with respect to consumption c_t and asset holdings A_{t+1}, that are carried into the next period. The maximization is performed subject to the following constraints:

$$c_t + A_{t+1} \leq X_t \tag{7}$$

$$X_{t+1} = A_{t+1}(1+r) + I_{t+1}Y_{t+1} + (1-I_{t+1})P(\overline{Y}_{t+1}) \quad (8)$$

$$\overline{Y}_{t+1} = \begin{cases} \frac{t\overline{Y}_t + \tilde{Y}_{t+1}}{t+1} & \text{if } t < T^{ret} \\ \overline{Y}_t & \text{if } t \geq T^{ret} \end{cases}$$

The first inequality is a standard budget constraint that tells us that consumption plus assets carried into the next period cannot exceed current cash-on-hand. The second equality is the law of motion of cash-on-hand. Cash-on-hand in the next period is given by the assets carried into the next period augmented by the net interest rate earned, plus non financial income. It is understood that if the indicator function $I_{t+1} = 1$ then the agent is working and earns net labor income Y_{t+1}, while if $I_{t+1} = 0$ the agent is retired and collects social security benefits $P(\overline{Y}_{t+1})$. The last equation represents the law of motion of average past gross earnings that enter the calculation of the pension benefits. Gross earnings at age t are denoted $\tilde{Y}$ and are obtained from net earnings Y_t by way of a suitable tax function $\tau(\tilde{Y}_t)$. Finally, the maximization is subject to the stochastic earnings processes defined in the previous subsection, and to the borrowing constraint $A_{t+1} \geq \underline{A}$.

3 Calibration

The model period is taken to be one year. Agents enter the labor market, hence the model, at age 20, retire at age 65 and die for sure at age 100. Before that age, the probability of survival from one year to the next are taken from the Berkeley Mortality Database. With respect to preference parameters we first perform a set of experiments for different values of the elasticity of inter-temporal substitution and risk aversion. We then move to a set of experiments where we search the values of risk aversion and the elasticity of inter-temporal substitution so that the distance between the model and data wealth profile over the working life is minimized. In each case we set β, the subjective discount factor, so that the average wealth-to-income ratio is equal to 2.5. While at first sight this value is lower than the one in the aggregate data, in practice it reflects correctly the wealth-to-income ratio in the bottom 95 percent of the earnings distribution in the PSID.[2] This is the part of the population we

[2]While the best source for data about wealth is the Survey of Consumer Finances (SCF), as pointed out by Bosworth and Anders (2008), the two data sets generate very similar results once the top 5 percent

are interested in given that the empirical estimates of the insurance coefficients are based on the PSID and CEX, which are well known not to represent accurately the top of the distribution.

For the deterministic common component of the labor income process we take a third order polynomial in labor market experience, that is, age minus 20, and use the coefficients estimated by Cocco *et al.* (2005). As for the stochastic component of earnings, we have to assign three parameters, that is, the variance of the permanent and temporary shocks η and ε and the initial variance of the permanent shock $\sigma^2_{z_0}$. We give σ^2_η a value of 0.01 to match the increase in earnings dispersion over the life-cycle observed in PSID data and we assign a value of 0.05 to σ^2_ε based on the point estimate by Blundell *et al.* (2008). Finally, we set $\sigma^2_{z_0}$ to 0.15 so as to match earnings dispersion at age 25.[3]

With respect to assets we set an interest rate of 3.5 percent. We do not determine the interest rate in equilibrium since the model is not meant to capture the behavior of households in the top of the wealth distribution who hold a disproportionate share of total wealth and, hence, are key in determining the equilibrium value of returns. Assets can be held subject to a no-borrowing constraint in the benchmark case but we also experiment with an alternative case where the agents may borrow up to the natural borrowing limit.

We model social security benefits so as to mimic the actual US system. In order to do that, we need to compute the average gross earnings over the lifetime of the agent and then to apply a formula that converts that average into a gross pension benefit. The formula for the US that we apply assigns a 90 percent replacement ratio for earnings up to 18 percent of average, a 32 percent replacement ratio from this bend point to next one, set at 110 percent of average earnings, and finally a 15 percent replacement ratio for earnings above 110 percent average earnings. Finally, we scale the benefits up so that the replacement ratio for the average earner is 45 percent.

Given that in our model the earnings process is based on net earnings, while in the US social security system the benefit formula is computed based on average gross earnings, we need to back out gross earnings from our model net earnings. To do that we invert the progressive tax function formula estimated by Gouveia and Strauss (1994) and now widely used in macroeconomics. If we denote the tax function with the letter τ and gross earnings

wealthiest households are removed.

[3]Overall, these are the values used in the benchmark calibration by Kaplan and Violante (2010).

of individual i at time t by $\tilde{Y}_{i,t}$ the cited tax function takes the form:

$$\tau(\tilde{Y}_{i,t}) = \tau^b[\tilde{Y}_{i,t} - (\tilde{Y}_{i,t}^{-\tau^\rho} + \tau^s)^{-\frac{1}{\tau^\rho}}] \tag{9}$$

To attribute values to the parameters of this function, we follow Kaplan and Violante (2010) and set $\tau^b = 0.258$ and $\tau^\rho = 0.768$ from the original work of Gouveia and Strauss (1994) and then set τ^s so that the ratio of personal income tax receipts to labor income is about 25 percent like in the US. With the tax function fully defined it is possible to recover gross earnings from net earnings by solving the equation: $\tilde{Y}_{i,t} - \tau(\tilde{Y}_{i,t}) = Y_{i,t}$. The tax function described above is then also used on 85 percent of gross social security benefits to get net benefits.

4 Results

In this section we report the results of the quantitative analysis of the model. We first perform an extensive exploration of the parameter space. Initially we specialize the Epstein-Zin preferences to the usual expected utility case by setting α=γ. In this case we consider values of risk aversion of 2, 5, 10, 15 and 20. Then for each risk aversion case we solve again the model for values of the elasticity of inter-temporal substitution of 0.5, 0.8 and 1.25. We report the results both for the model with a zero borrowing constraint and for the opposite case of the natural borrowing limit. Finally, in light of the lessons learned with this analysis we report the results of the model solved under a preferred calibration where preference parameters are chosen so as to minimize the distance between the profile of asset accumulation during the working part of the life-cycle in the model and in the data.

We report values of the insurance coefficients of both the permanent and the temporary shock, although our focus will be on the former given the finding of Kaplan and Violante (2010) that these are the ones that the standard incomplete market model has a hard time to explain. Given our focus on exploring a solution to the inability of the model to match the data, we will focus on the model counterpart of the empirically estimated coefficients.[4]

[4]Having a model at hand, one can also compute the true insurance coefficients and use them to study the magnitude of the bias of the BPP estimator, however this is outside the scope of the present research. See Kaplan and Violante (2010) for a discussion of the source of the estimation bias and when it is most likely to be greater.

Before moving to the actual description of the results in the next subsection, we will briefly describe how the insurance coefficients are defined and computed.

4.1 BPP insurance coefficients

Let $y_{i,t}$ and $c_{i,t}$ be the log deviation of net labor income and consumption from their respective deterministic life-cycle trend. In general, the deviation of income can be the result of different shocks that we can generically denote $x_{i,t}$. Following Kaplan and Violante (2010) we can define the insurance coefficient for shock $x_{i,t}$ as:

$$\phi^x = 1 - \frac{cov(\Delta c_{i,t}, x_{i,t})}{var(x_{i,t})} \tag{10}$$

If the received shock translated one-to-one into a change in consumption ϕ^x would be equal to 0, while in the opposite case where consumption did not react at all to the shock the index would be equal to 1. The index then is a measure of the proportion of the shock that is not translated into consumption growth and, hence, is a measure of the extent to which shocks are insured, with a higher value corresponding to better insurance. Having data on both consumption and the shock, as it happens in a model simulation, one can directly compute the true value of the index. Alternatively, given the earnings process described in the previous sections and used in much of the quantitative macroeconomics literature the coefficients can be estimated from income and consumption data alone provided the following two identifying restrictions are assumed:

$$cov(\Delta c_{i,t}, \eta_{i,t+1}) = cov(\Delta c_{i,t}, \varepsilon_{i,t+1}) = 0 \tag{11}$$

and

$$cov(\Delta c_{i,t}, \eta_{i,t-1}) = cov(\Delta c_{i,t}, \varepsilon_{i,t-2}) = 0 \tag{12}$$

The two assumptions state that consumption growth can be correlated neither with future nor past shocks.[5] Under these assumption it can be shown that:

$$-\, cov(\Delta y_{i,t}, \Delta y_{i,t+1}) = var(\varepsilon_{i,t}) \tag{13}$$

$$-\, cov(\Delta c_{i,t}, \Delta y_{i,t+1}) = cov(\Delta c_{i,t}, \varepsilon_{i,t}) \tag{14}$$

[5] Given this identifying assumption, Epstein-Zin preferences are another source of bias, however assuming that the empirical data are generated by household that have these preferences, the application of the BPP estimator to both data and model would introduce the same kind of bias making the comparison legitimate.

which allows the econometrician to identify ϕ^{ε} and

$$-\,cov(\Delta y_{i,t}, \Delta y_{i,t-1} + \Delta y_{i,t} + \Delta y_{i,t+1}) = var(\eta_{i,t}) \tag{15}$$

$$-\,cov(\Delta c_{i,t}, \Delta y_{i,t-1} + \Delta y_{i,t} + \Delta y_{i,t+1}) = cov(\Delta c_{i,t}, \eta_{i,t}) \tag{16}$$

that identifies ϕ^{η}, the insurance coefficients for the permanent shock. Failure of the given assumptions leads to biased estimates that can be assessed when using model simulated data.[6]

4.2 The expected utility case

We report in Table 1 the insurance coefficients obtained from the model simulation when preferences are assumed of the standard expected utility form for risk-aversion coefficients ranging from 2 to 20. Looking at the first row, we see that the empirical estimates, taken from Blundell *et al.* (2008), are 0.36 for permanent shocks and 0.95 for transitory shocks. In the case of transitory shocks, the estimates based on simulated data are always very close to their data counterparts, ranging from 0.885 in the case of risk aversion of 2, to about 0.90 for risk aversion of 10 or more. The picture changes radically as far as the insurance coefficients for permanent shocks are concerned. The estimated coefficient is 0.167 for risk aversion of 2 and it rises up to 0.266 for risk aversion of 20. While this increase is substantial, it still leaves the coefficient 0.1 points below the empirical value. The reason for the increase is that risk-averse agents dislike consumption volatility more and, hence, in the face of positive shocks they will save a larger part of them in order to finance consumption when the bad shock hits. This increase in savings also has another effect that can be seen in the last column of Table 1: in order for the model to still match the targeted level of the wealth-to-income ratio, it is necessary to reduce the subjective discount factor from 0.975 to 0.545, clearly a value that is well outside of what is acceptable.

4.3 The Epstein-Zin Case

It is well known that Epstein-Zin preferences allow the model to fully disentangle risk aversion from inter-temporal substitution. In this subsection we thus exploit this increased

[6] The introduction in the text is a basic description of the parameters of interest that we compute in the model experiments. For a thorough introduction to the estimation of the insurance coefficients see Blundell *et al.* (2008) and Kaplan and Violante (2010) upon which our treatment of the issue is based.

Table 1: Insurance coefficients by risk aversion (Expected utility)

	Permanent shock	Transitory shock	β
Data	0.36	0.95	
ra = 2	0.167	0.885	0.975
ra = 5	0.170	0.892	0.936
ra = 10	0.212	0.904	0.822
ra = 15	0.238	0.902	0.675
ra = 20	0.266	0.900	0.545

Table 2: Insurance coefficients by risk aversion (Epstein-Zin)

	Permanent shock	Transitory shock	β
Data	0.36	0.95	
EIS = 0.5			
ra = 2	0.167	0.885	0.975
ra = 5	0.186	0.887	0.947
ra = 10	0.267	0.902	0.925
ra = 15	0.314	0.897	0.91
ra = 20	0.340	0.889	0.897

freedom in choosing preference parameters to check if it is possible to improve the ability of the model to match the empirical insurance coefficients. We proceed in two steps. First, in Table 2 we keep the elasticity of inter-temporal substitution fixed at 0.5 and consider the usual values of risk aversion in the range 2 to 20. As can be seen in the second column, the estimated insurance coefficient for the permanent shock raises from 0.167 when risk aversion is 2 to 0.34 when risk aversion is 20. The latter value is already quite close to 0.36, the measure found in the data. As it can be seen in the last column of Table 2, this can be obtained with a substantially smaller decrease in the value of the subjective discount factor. The value of β that is needed to match to targeted wealth-to-income ratio is 0.975 when risk aversion is 2 and declines only to 0.897 when risk aversion is 20, a value that while still smaller, it is not very far from what macro-economists think plausible.

In the second step, we alternatively proceed by fixing risk aversion at a value of 10 and checking how results change when the elasticity of inter-temporal substitution is raised from

Table 3: Insurance coefficients by EIS (Baseline)

	Permanent shock	Transitory shock	β
Data	0.36	0.95	
ra=10			
EIS = 0.1	0.212	0.904	0.822
EIS = 0.5	0.267	0.902	0.925
EIS = 0.8	0.287	0.901	0.935
EIS = 1.25	0.317	0.899	0.942

the corresponding expected utility value of 0.1 to 1.25. Looking at the second column in Table 3, we can notice that even for constant risk aversion an increase in the elasticity of inter-temporal substitution brings about a substantial increase in the estimated coefficient for permanent shocks, from 0.212 to 0.317. Also, looking at the last column of Table 3 we see that this is obtained with a contemporaneous increase in the required value of the subjective discount factor, from 0.822 when the elasticity of inter-temporal substitution is 0.1 to 0.942 when it is 1.25. The latter is clearly a value that is already close to accepted values in macroeconomic modelling.

Finally, in Table 4 we put together the insights obtained in the previous analysis and consider a broad range of parameter values including for risk aversion the values of 2, 5, 10, 15 and 20 and for each of them setting the elasticity of inter-temporal substitution at 0.5, 0.8 and 1.25. What the table shows is that for certain combinations of the elasticity of inter-temporal substitution and risk aversion the model goes a long way towards rationalizing the observed empirical values of the insurance coefficients of permanent shocks. For example, for the combination of EIS of 0.8 and risk aversion of 20 the insurance coefficient is 0.366 and the targeted wealth-to-income ratio is obtained for β set to 0.917, while if we are willing to accept a value of the EIS of 1.25 we can get an insurance coefficient of 0.369 for risk aversion equal to 15 and a subjective discount factor of 0.934.

In order to briefly conclude this section, we also want to point out at the results concerning the estimated insurance coefficients for temporary shocks. With the exception of the cases with risk aversion set to 2 and with an elasticity of inter-temporal substitution of 0.8 or 1.25, where the insurance coefficients declines to 0.863 and 0.790 respectively, the insurance coefficients for temporary shocks remain in the narrow range between 0.89 and

Table 4: Insurance coefficients by risk aversion and EIS (Baseline)

	Permanent shock	Transitory shock	β
Data	0.36	0.95	
EIS = 0.5			
ra = 2	0.167	0.885	0.975
ra = 5	0.186	0.887	0.947
ra = 10	0.267	0.902	0.925
ra = 15	0.314	0.897	0.91
ra = 20	0.340	0.889	0.897
EIS = 0.8			
ra = 2	0.135	0.863	0.963
ra = 5	0.201	0.890	0.950
ra = 10	0.287	0.901	0.935
ra = 15	0.337	0.894	0.925
ra = 20	0.366	0.887	0.917
EIS = 1.25			
ra = 2	0.027	0.790	0.951
ra = 5	0.212	0.890	0.950
ra = 10	0.317	0.899	0.942
ra = 15	0.369	0.891	0.934
ra = 20	0.394	0.881	0.928

0.90 values that are very close to the empirical estimates.

4.4 Interpretation

In this section we describe the mechanism that generates our results. Models like the one considered here exhibit precautionary savings, which is the dominant factor for accumulating wealth in the initial part of the life-cycle.[7] As risk aversion increases, households dislike more consumption fluctuations, hence they will save a larger proportion out of positive shocks to use those savings to insulate consumption from negative earnings shocks. As a consequence, we observe both an increase in savings early in the life-cycle and an increase in the observed insurance coefficients. This effect is common both to the model with expected utility and to the model with Epstein-Zin preferences. The increase in wealth accumulation for precautionary reasons, given the calibration constraint on the wealth-to-income ratio, implies the need to reduce the value of the subjective discount factor. Where the two preference specifications differ is with respect to savings late in the working life. With expected utility, raising risk aversion implies reducing the elasticity of inter-temporal substitution, which is connected to the former by an inverse relationship. A lower elasticity of inter-temporal substitution though, leads to higher saving in mid-life because the agents want a flatter consumption profile, hence they need more wealth for the retirement period. As a consequence, the extra wealth accumulation is more limited in the Epstein-Zin case than in the expected utility case and the subjective discount factor needs not be reduced so much as becoming unrealistically low to support a substantial raise in risk aversion at constant wealth-to-income ratios. Moreover, as it can be seen in Table 3, the insurance coefficient for the permanent shock increases when the elasticity of inter-temporal substitution increases even in the absence of any increase in risk-aversion. The intuition is similar. With a higher elasticity of inter-temporal substitution the household is willing to accept a more downward sloping consumption profile late in life, hence it will save less out of late working age income. Given the constant wealth-to-income ratio required by the calibration, this allows the model to accept a higher value of β. In turn, this raises savings early in life. Savings early in life is also increased because the tension between anticipating consumption in the face of an upward sloping earnings profile and delaying it to accumulate precautionary wealth is more

[7] See Gourinchas and Parker (2002).

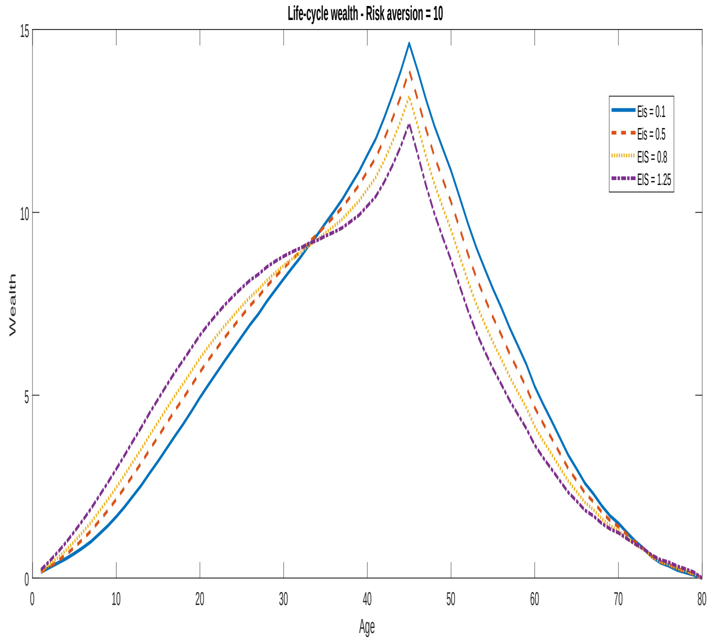

Figure 1: Life-cycle profiles of wealth.

easily resolved in favor of the latter if the agent is sufficiently elastic. In summary, having the possibility to keep the elasticity of inter-temporal substitution high implies that wealth is reshuffled from mid-life, when there is more than enough to insure shocks, to early life when insurance is poor.

This is confirmed by Figure 1, which reports the life-cycle profile of wealth for parameterizations of the model with a constant risk aversion, set equal to 10, and increasing values of the elasticity of inter-temporal substitution, ranging from 0.1 (expected utility case) up to 1.25. Figure 1 shows that the age profile of wealth during working life is convex shaped in the expected utility case. When the elasticity of inter-temporal substitution is progressively raised, it changes to a convex-concave shape that gives it a distinct hunchbacked profile. In the expected utility case wealth after 20 model periods is roughly one third of peak wealth around retirement age, while in the model with an elasticity of substitution of 1.25 it is about 53 percent of peak wealth. The consequences for the insurance coefficients are explored in Table 5, where we report the insurance coefficients of the permanent shocks for the same parameterizations represented in Figure 1. As it can be seen, the substantial reduction in peak wealth caused by the increase in the elasticity of inter-temporal substitution barely affects insurance coefficients near the end of the working life, which remain confined in a narrow

Table 5: Estimated coefficients by age groups (Permanent shock)

Age group	27-31	57-61
ra=10		
EIS = 0.1	0.029	0.615
EIS = 0.5	0.175	0.627
EIS = 0.8	0.162	0.627
EIS = 1.25	0.198	0.617

range between 0.615 and 0.627. On the other hand, the larger wealth accumulation early in life raises the insurance coefficients in a substantial way for the age group between 27 and 31: the coefficient increases from 0.029 when the elasticity of inter-temporal substitution is 0.1 to 0.198 when it is 1.25.

The above analysis, beside providing insights into why Epstein-Zin preferences allow the model to get closer to matching empirical coefficients, also points to another benefit of this choice. In fact, according to Blundell *et al.* (2008) estimates, the insurance coefficients for permanent shocks do not show any trend with age, while as shown in Kaplan and Violante (2010) the standard expected utility model generates strongly increasing and convex insurance coefficients by age.[8] The analysis conducted in this paper though, shows that increasing the elasticity of inter-temporal substitution makes one step in the correct direction by flattening the life-cycle profile of the coefficients. Results are even starker for the case of a risk aversion of 20, the highest value considered here. In that case, combined with an elasticity of inter-temporal substitution of 1.25, we get an estimated insurance coefficient for permanent shock that is 0.342 for the age group 27 to 31 and 0.595 for the age group 57 to 61.[9]

4.5 The natural borrowing limit case

The models solved so far have assumed that agents cannot borrow. In this subsection we repeat the same experiments that we performed before but in a version of the model

[8] We get their same results when using expected utility with low risk aversion, which we do not report for the sake of brevity.

[9] For the sake of space we do not report the full set of tables with the insurance coefficients by age groups but they are available upon request.

Table 6: Insurance coefficients for permanent shock (Model with debt)

	Permanent shock	Transitory shock	$\%W < 0$	β
EIS = 0.5				
ra = 2	0.261	0.943	0.274	0.979
ra = 5	0.274	0.931	0.212	0.950
ra = 10	0.326	0.915	0.147	0.928
ra = 15	0.354	0.906	0.108	0.913
ra = 20	0.382	0.899	0.115	0.902
EIS = 0.8				
ra = 2	0.253	0.94	0.342	0.966
ra = 5	0.279	0.928	0.194	0.951
ra = 10	0.333	0.913	0.097	0.937
ra = 15	0.373	0.903	0.112	0.927
ra = 20	0.405	0.896	0.124	0.920
EIS = 1.25				
ra = 2	0.184	0.909	0.551	0.955
ra = 5	0.288	0.924	0.175	0.952
ra = 10	0.355	0.908	0.098	0.943
ra = 15	0.403	0.898	0.120	0.936
ra = 20	0.438	0.891	0.139	0.930

where households can borrow, subject only to the constraint that they can repay for sure their debts by the time they reach the maximum possible age or the so called natural borrowing limit. Results are reported in Table 6 for all the risk aversion and elasticity of inter-temporal substitution values that we considered in the zero borrowing limit case. Looking at the first column of the table, we see that introducing debt further increases the estimated insurance coefficients for permanent shocks. As it can be seen in the top panel, in this case it is possible to reach an insurance coefficient of 0.354, already very close to the empirical one, at a value of risk-aversion of 15 in the case where the elasticity of inter-temporal substitution is 0.5. When the elasticity of substitution is raised to 0.8, the empirical value of the coefficients is reached somewhere between risk aversion of 10 and 15, while looking at the bottom panel we see that when the elasticity of inter-temporal substitution is 1.25 a coefficient of risk-aversion of 10 generates an estimated coefficient of 0.355. For this parametrization a value of the subjective discount factor of 0.943 supports a wealth-to-income ratio of 2.5. The insurance coefficients for temporary shocks increase as well, albeit to a lesser extent. Looking at the third column of the table, we can also see that the fraction of agents with negative wealth ranges from a maximum of 0.551 to a minimum of 0.097. For the parameterizations whose associated insurance coefficients are consistent with the empirical ones though, the fraction of agents with negative wealth is always close to 10 percent. This value falls in the range of 5.8 percent to 15 percent reported by Huggett (1996) from the Survey of Consumer Finances.[10] Overall, we can then say that adding debt further improves the fit of the model to the data. The main reason is that when households may hold debt they can drive wealth into negative territory when faced with negative shocks, thus improving their insurance opportunities. Moreover, this reduces wealth accumulation overall, allowing the model to support values of β that are higher and hence closer to standard macroeconomic practice, although this effect is quite small.

4.6 Extensions

In the current section we consider three variations on the basic model. These include a model where agents start life with non-zero wealth, one where a fraction of the agents

[10] The smallest figure refers to a measure of net worth that includes durable goods like cars, while the largest figure does not include them.

have defined benefits pensions on top of social security payments and finally one where the earnings process shows very persistent but not fully permanent shocks. The models are first solved in the zero borrowing constraint case and then in the natural borrowing limit case. In all cases the subjective discount factor is adjusted so as to keep the wealth-to-income ratio constant at the baseline target value. Results are reported in Table 7 and 8. For the sake of brevity we only report the estimated insurance coefficients against a permanent shock and the subjective discount factor associated with the given parametrization.

The first two columns of Table 7 report the results concerning the model with non-zero initial wealth. Wealth at the beginning of life is assumed to be log-normally distributed with the mean and standard deviation chosen so as to match the one measured in the PSID for the age group 20-25. As we can see the insurance coefficients increase with respect to the baseline case, however the increase is not big, at most 0.04 in the case of an elasticity of inter-temporal substitution of 1.25 and risk aversion equal to 2. This said the increase is declining to 0.01 when risk aversion grows and the insurance coefficients in the baseline model approach the empirical level. The interpretation is that starting life with positive wealth allows agents to be better insured at a point of the life-cycle when wealth would be very low and hence insurance would be poor, however as risk-aversion increases precautionary savings becomes substantial very early in life, giving a small extra value to the inherited wealth.

Next we turn to the model with defined benefit pensions. In order to calibrate pensions we use data reported in Scholz *et al.* (2006). The authors report data on median earnings and on median defined benefit wealth by deciles of the life-time earnings distribution in their sample from the Health and Retirement Study. Using our average past earnings distribution at retirement age we similarly partition it into deciles. We then attribute to each cell a pension benefit such that the ratio of its expected present value at retirement to median earnings in the model matches the data in the above mentioned paper. This calibration is clearly a simplification for several reasons. First in partitioning agents at retirement, the concept of average past earnings although very similar is not exactly the same as that of present value of earnings.[11] Second the only uncertainty about whether an agent will be assigned a defined pension and its level is related to the unfolding of the earnings realizations over the life-cycle. In reality agents may cycle through different jobs that may or may not

[11]The two may differ because of the distribution of shocks over the life-cycle, however the correlation of the two measures is very high.

Table 7: Insurance coefficients: Other models - ZBC

	$W_0 > 0$		DB pensions		AR(1) Earnings	
	Model BPP	β	Model BPP	β	Model BPP	β
EIS = 0.5						
ra = 2	0.196	0.973	0.194	0.981	0.226	0.970
ra = 5	0.218	0.945	0.221	0.953	0.257	0.941
ra = 10	0.286	0.923	0.292	0.932	0.332	0.916
ra = 15	0.330	0.908	0.333	0.917	0.373	0.899
ra = 20	0.356	0.895	0.360	0.904	0.399	0.886
EIS = 0.8						
ra = 2	0.169	0.962	0.172	0.967	0.201	0.960
ra = 5	0.232	0.949	0.233	0.954	0.269	0.945
ra = 10	0.306	0.934	0.314	0.940	0.352	0.929
ra = 15	0.349	0.923	0.356	0.930	0.396	0.917
ra = 20	0.376	0.914	0.383	0.921	0.422	0.908
EIS = 1.25						
ra = 2	0.069	0.951	0.072	0.954	0.109	0.950
ra = 5	0.242	0.950	0.248	0.953	0.284	0.948
ra = 10	0.332	0.941	0.342	0.945	0.382	0.937
ra = 15	0.379	0.933	0.385	0.937	0.426	0.929
ra = 20	0.404	0.926	0.413	0.931	0.453	0.922

offer defined benefit pension plans independently of the earnings shock. Our approach though, beside avoiding the computational burden of adding a further state variable with potentially as many realizations as there are working years, it allows us to capture the median replacement ratio for defined benefit pensions and the fact that since these are increasing in lifetime earnings they tend to undo the insurance element intrinsic to social security.[12] As far as the results are concerned, the third and fourth column of Table 7 show that the insurance coefficients increase and the subjective discount factor increases compared to the baseline case. The insurance coefficients increase by an amount between 0.02 and 0.045 while the subjective discount factor increases by about 0.005 for all parameterizations. The intuition is that having defined benefit pensions increases the effective replacement ratios, especially for agents with higher lifetime earnings, thus reducing the need to accumulate wealth in mid-life for the retirement age. Given the constraint imposed by matching the wealth-to-income target this requires an increase in the subjective discount factor. At the same time it also implies a redistribution of wealth from mid-age to the beginning of the life-cycle when insurance is poor, increasing the overall coefficient. As in the case of non-zero initial wealth this effect is smaller for higher values of risk-aversion.

Finally, we consider a case with AR(1) persistent shocks. For the parameters of the process we follow the estimates of Guvenen (2009) and set the autocorrelation coefficient to 0.988, the standard deviation of the innovation to 0.015 and the standard deviation of the temporary shock to 0.061. We retain though the standard deviation of shocks at the beginning of life of 0.15 of the baseline model. Results are reported in the fifth and sixth column of Table 7. The insurance coefficients increase by a substantial amount, that is, about 0.06 compared to the case of permanent shocks. Under this process hence, it is possible to match the empirical insurance coefficients with risk aversion of about 10 or slightly more when the elasticity of inter-temporal substitution is 0.5 or 0.8, while a coefficient of risk aversion of less than 10 is sufficient when the elasticity of inter-temporal substitution is 1.25. The subjective discount factor is in all cases slightly lower than in the baseline. The interpretation of these results is that under a highly persistent but not

[12]Based on Scholz *et al.* (2006) data in fact, our pensions are zero in the bottom three deciles of the average past earnings distribution and then they show a monotonically increasing replacement ratio in the remaining ones. For a more detailed modelling of defined benefit pensions one can see Zhou and MacGee (2014).

Table 8: Insurance coefficients: Other models - NBC

	DB pensions			AR(1) Earnings		
	Model BPP	β	$\%W < 0$	Model BPP	β	$\%W < 0$
EIS = 0.5						
ra = 2	0.271	0.984	0.206	0.329	0.975	0.33
ra = 5	0.288	0.955	0.154	0.340	0.944	0.246
ra = 10	0.339	0.935	0.100	0.397	0.921	0.168
ra = 15	0.368	0.920	0.084	0.425	0.905	0.137
ra = 20	0.395	0.909	0.102	0.451	0.893	0.142
EIS = 0.8						
ra = 2	0.264	0.969	0.268	0.319	0.964	0.39
ra = 5	0.296	0.955	0.147	0.347	0.947	0.22
ra = 10	0.348	0.941	0.075	0.404	0.932	0.135
ra = 15	0.387	0.932	0.096	0.450	0.922	0.146
ra = 20	0.417	0.924	0.118	0.476	0.914	0.152
EIS = 1.25						
ra = 2	0.213	0.956	0.50	0.259	0.953	0.549
ra = 5	0.309	0.954	0.142	0.356	0.948	0.198
ra = 10	0.374	0.946	0.082	0.429	0.939	0.130
ra = 15	0.417	0.939	0.114	0.477	0.932	0.149
ra = 20	0.448	0.934	0.141	0.508	0.926	0.166

fully permanent process it is easier to insure shocks. This gives the incentive to accumulate more wealth pushing down the discount factor that is needed to match the wealth-to-income target.[13]

Table 8 shows the results in the natural borrowing limit case. For the sake of space and given their limited value added, we omit those concerning the model with non-zero initial wealth. The first three columns report the case where the natural borrowing limit

[13]As it was explained in Kaplan and Violante (2010), when shocks are persistent an additional source of bias is introduced in the estimates of the insurance coefficients. However applying the BPP procedure to model simulated data introduces the same bias that is introduced in the data if the data are actually generated by an AR(1) process, hence applying the BPP estimated coefficients to model simulated data and comparing them to the empirical one is still correct.

is interacted with defined benefit pension plans. Adding the latter increases the insurance coefficients against permanent shocks by about 0.015 and increases the subjective discount factor by about 0.007. By doing this the insurance coefficient for the models with elasticity of inter-temporal substitution of 0.5 and 0.8 and risk aversion of 10 are about 0.34 and 0.35 respectively, both very close to the empirical value of 0.36. This result is obtained with values of the subjective discount factor in the region of 0.94.

The last three columns of Table 8 show the results obtained when the earnings process in Guvenen (2009) is considered jointly with the natural borrowing limit. As it can be seen, in this case, values of the insurance coefficients of around 0.35, close to the empirical level, are obtained for risk-aversion coefficients of 5 and all of the three values of the inter-temporal elasticity of substitution considered. Given this, for higher values of risk aversion the empirical insurance coefficient is easily exceeded. While this might suggest that a small mis-specification in the earnings process can lead the model to generate the same insurance coefficients as those in the data for low value of risk aversion, looking at the last column shows that this comes at the cost of having agents resort to debt too frequently: in the cases of risk aversion of 5 and elasticity of inter-temporal of 0.5 or 0.8, the share of agents with negative wealth is above 20 percent, well in excess of the empirical value. The intuition is that shocks that are very persistent are better insurable than fully persistent ones, making the agents more willing to take on debt to smooth consumption.

4.7 Discussion

The results so far obtained, showed that the flexibility of Epstein-Zin preferences allows the model to match the average estimated insurance coefficient for permanent shocks with values of the subjective discount factor that are in line with what is accepted in economics. The inspection of the mechanism also showed that this is obtained by shifting wealth mainly from pre-retirement age, when agents are already well-insured, to young age when this is not the case. Here we want to briefly discuss the plausibility of this mechanism with respect to two issues, that is, first the extent to which the shape of the wealth profiles that we obtained under the best Epstein-Zin parameterizations are empirically acceptable and, second the acceptability of the values of γ and α that we have used in this study.

We start with the first issue that is explored in Figure 2 and Table 9. Figure 2 reports

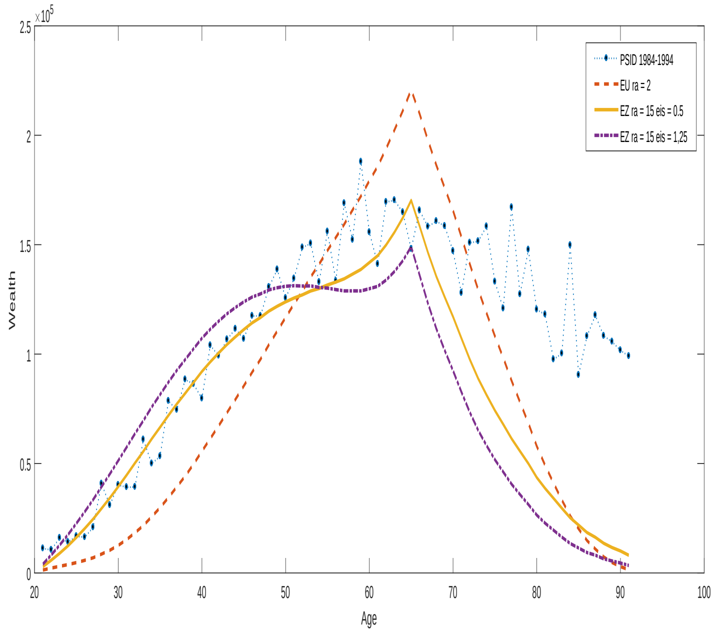

Figure 2: Life-cycle profiles of wealth.

empirical wealth profiles together with a selection of life-cycle wealth profiles simulated from different model parameter choices. Simulated profiles are converted to dollar values by re-scaling them so that model and data average wealth match. The empirical wealth profiles are based on PSID wealth data obtained by averaging the profiles for the years 1984, 1989 and 1994, all converted to 2000 dollars and removing the top 5 percent observations so that they reflect the same subset of the population that the model is meant to capture. The chosen model profiles correspond to the baseline case of expected utility with risk-aversion equal to 2, a model with Epstein-Zin preferences where the elasticity of inter-temporal substitution is kept at 0.5 while risk-aversion is raised to 15 and a model where, while keeping risk aversion to 15, we raise the elasticity of inter-temporal substitution to 1.25. In all cases the parameterizations considered refer to models with the strict no borrowing constraint. As the comparison between the dashed line and the dotted line shows, the baseline expected utility model substantially underestimates wealth at young ages while grossly overestimating it in the years prior to retirement. On the other hand, the model with elasticity of inter-temporal substitution equal to 0.5 and risk aversion equal to 15, represented by the continuous line, does a good job at following the empirical life-cycle profile of wealth during working life. Engineering a further transfer of wealth towards younger ages, in this case by further raising the elasticity of inter-temporal substitution, leads to a more hunch-backed profile of wealth that in this case sightly overestimates the empirical pattern at younger ages, as shown by the dashed-dotted line. We remark that the parametrization of the continuous line corresponds to an insurance coefficient for the permanent shock of 0.314, already very close to the empirical estimate in BPP, while the dashed-dotted line represents a parametrization that generates an insurance coefficient of 0.369, even larger than the empirical estimate.

To make this reasoning more precise, in Table 9 we report the minimum squared distance between the model and data life-cycle profiles. The statistics is computed taking into account wealth during working age only, given that this is the relevant one for the insurance coefficients. The table shows that for any level of the elasticity of substitution the minimum value of the minimum squared distance between the model and data wealth is reached for values of risk aversion of 10 or 15. Looking back at Table 4 and 6, one can see that in all cases the corresponding insurance coefficients are in the range of 0.3 and above. The absolute minimum squared distance is reached in the model with debt, elasticity of substitution of 1.25 and risk aversion of 10. The insurance coefficient for permanent shocks in this parametrization is 0.355, with a subjective discount factor of 0.943 very close to common values used in macroeconomics. Overall, the above analysis suggests that the parameterizations allowed by Epstein-Zin preferences improve the ability of the model to explain the empirically observed insurance coefficients for permanent shocks by redistributing wealth over the life-cycle in a way that makes the life-cycle profile of wealth of the artificial economy closer to the one in the data.[14]

The remaining part of the discussion concerns preference parameters, that is, the coefficient of relative risk aversion and the elasticity of inter-temporal substitution. With respect to risk aversion, the model can deliver the right amount of insurance against permanent shocks with values that, depending on the elasticity of inter-temporal substitution range from 10 to 20. These values are higher than what is normally assumed in macroeconomic models, but it must be said that the key reason for assuming a low risk aversion is that a reasonable behavior of macroeconomic quantities hinges upon a relatively high elasticity

[14]Clearly, this is only one possibility to increase wealth accumulation in the first part of the life-cycle. Others might include accumulation for financing down-payment requirements to buy a house or to finance the children's college studies. It remains to be seen if under these alternative assumptions the result in terms of insurance coefficients would go through.

Table 9: Data — model wealth squared distances

Risk aversion	2	5	10	15	20
EU	4.49	4.08	1.88	0.90	0.75
EZ - No debt					
Eis = 0.5	4.49	3.78	1.07	0.80	1.35
Eis = 0.8	6.08	3.46	0.80	1.11	1.99
Eis = 1.25	12.09	3.26	0.79	1.96	3.37
EZ - Debt					
Eis = 0.5	7.06	5.07	1.53	0.77	1.11
Eis = 0.8	9.61	4.55	0.89	0.99	1.70
Eis = 1.25	20.86	4.00	0.75	1.78	2.97

of inter-temporal substitution which under expected utility is linked to the former by an inverse relationship. Such a link is not present in the case of Epstein-Zin preferences. If on the other hand one looks at the experimental evidence, the values of risk aversion considered here appear acceptable. For example, Barsky *et al.* (1997) find that about two thirds of their sample shows a risk aversion coefficient of 15, with the rest of the sample equally split between risk aversions of 7, 6 and 4. The value of 10 or slightly more that we needed in the case of elasticity of inter-temporal substitution of 1.25 and 0.8 respectively and the natural borrowing limit is then reasonable in light of that evidence. With respect to the elasticity of inter-temporal substitution, microeconomic estimates vary substantially. For example, using British data Attanasio and Weber (1993) find values between 0.3 and 0.7. It is also true that the values tend to increase with wealth: for example Vissing-Jørgensen and Attanasio (2003) estimate an interval ranging from 1 to 1.4 for the population of stockholders, which is wealthier than average. Overall then, the values used here are again within the limits suggested by the available empirical evidence.

4.8 Optimized wealth profiles

In the previous sections we studied the behavior of estimated insurance coefficients as a function of the elasticity of inter-temporal substitution and risk aversion under several model specifications. We found that for suitable combinations of the two parameters it was possible to match the empirically estimated coefficients and that parameterizations that performed well under the above criterion also generated life-cycle profiles of wealth that are closer to the empirical ones. In the current section we want to further follow this line of reasoning and search for the preference parameter combinations that minimize the distance between model and empirical life-cycle wealth profiles. We then check the insurance coefficients under these minimum distance parameterizations. More specifically as a distance criterion we use the minimum square distance of model and data wealth over the working part of the life-cycle. We make this restriction because shocks are received during working life, hence it is important to have a precise match of wealth during this portion of the life-cycle to make statements about the ability of the model to explain the level of insurance. Based on the results obtained thus far we restrict the search on the interval of risk-aversion coefficients between 5 and 20. We also restrict the elasticity of inter-temporal substitution to values between 0.3 and 0.8. The upper bound is so restricted because higher estimates especially above 1 are generally found for more wealthy individuals or stock-holders while on the one hand we calibrated the model using data that remove the top 5 percent wealthiest households and on the other hand we do not have stocks and hence a distinction between stockholders and non-stockholders. We solve several versions of the model, that is the baseline, the model with defined benefit pensions and the model that assumes that the persistent component of model earnings follows an AR(1) process with the parameters taken from Guvenen (2009). For all the three versions we present both results with the tight borrowing constraint and for the model with the natural borrow limit.

Before examining the results, Figure 3 reports the minimum squared distance between model and data wealth over the working life for the baseline model with zero borrowing constraint for the set of risk aversion and the elasticity of inter-temporal substitution that were searched over. As it can be seen the distance metrics shows a clear U-shaped pattern with respect to risk aversion with the minimum distance being reached for values between 10 and 15. Also the minimizing value of risk aversion falls as the elasticity of inter-temporal substitution increases.

Results for this experiment are shown in Table 10 where for the different versions of the model we report the estimated permanent and temporary insurance coefficient in the first and second column respectively, the percentage of agents with negative wealth in the third column and the values of risk aversion and the elasticity of inter-temporal substitution that minimize the distance of model and data wealth over working life in the fourth and

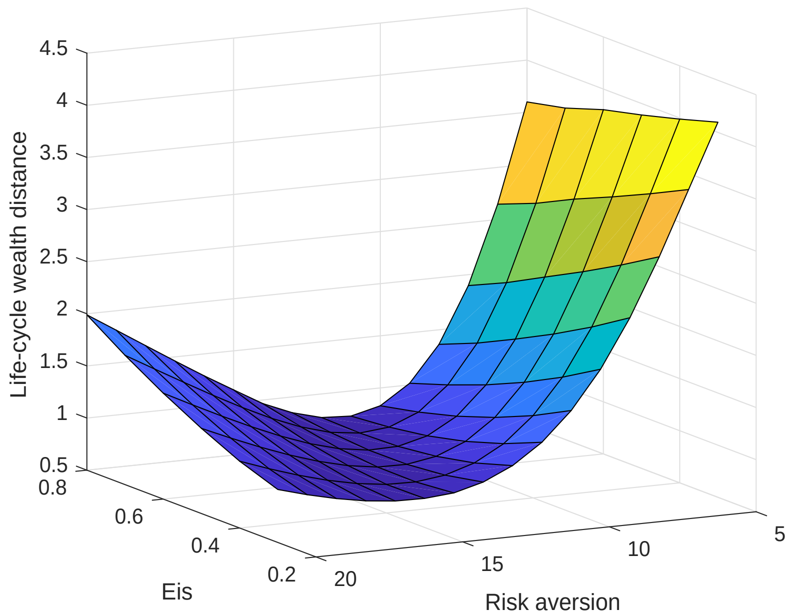

Figure 3: Life-cycle wealth squared distances.

Table 10: Estimated Insurance coefficients: Minimum distance parameters

	P-S	T-S	$\%W < 0$	ra	eis	β
Data	0.36	0.95				
ZBC						
Baseline	0.30	0.90	0.0	11	0.8	0.933
DB pensions	0.30	0.91	0.0	9	0.8	0.942
AR(1)	0.33	0.89	0.0	9	0.7	0.929
NBC						
Baseline	0.35	0.91	10.2	12	0.8	0.932
DB pensions	0.34	0.92	7.6	9	0.8	0.944
AR(1)	0.40	0.91	13.4	10	0.8	0.932

fifth columns. Finally, the last column reports the value of the subjective discount factor that allows each model to match the targeted wealth-to-income ratio of 2.5. What we can see from Table 10 is that for the baseline model with no borrowing the best fit wealth profile is obtained for the elasticity of inter-temporal substitution of 0.8 and a risk aversion coefficient of 11. The estimated coefficient for the permanent shock is 0.30 which is close to but slightly lower than the empirical value of 0.36. The estimated insurance coefficient for the temporary shock is 0.9, close to the empirical value. The subjective discount factor that allows the model to match the wealth-to-income target is 0.933 only slightly lower than what is used in macroeconomics. When borrowing is allowed in the form of the natural borrowing limit the estimated insurance coefficients rises to 0.35 which virtually matches the empirical value. This is obtained with a slightly larger value of risk aversion of 12. The increase in the insurance coefficients is directly explained by the fact that when debt is allowed agents can run wealth into negative territory to insure shocks. This would lower average wealth holdings early in life compared to the model with no borrowing, hence the increase in the risk aversion coefficient that is needed to match the wealth profile. The percentage of agents with negative wealth is 10.2 percent, within the values reported by Huggett (1996) and cited in previous sections of this work.

Interestingly, when defined benefit pensions are introduced there in no improvement in the estimated insurance coefficient and in the natural borrowing limit case there is even a small decrease in the permanent one. The minimizing value of the elasticity of inter-temporal substitution remains at 0.8 while that of risk aversion falls to 9 for both the model with no borrowing and the model with the natural borrowing limit. The subjective discount factor rises to 0.944. The interpretation is that with defined benefit pensions the replacement ratio would increase, leading to a higher discount factor to match the average wealth-to-income ratio. However, this also leads to relatively higher wealth accumulation early in life. Since the whole profile of wealth is now constrained this requires reducing risk aversion so as to reduce precautionary savings, which takes places early in the life-cycle, to compensate. The overall effect on the insurance coefficient against permanent shocks may then even be negative. This overturns the results of the previous sections where only the average wealth-to-income ratio was matched.

Finally, in the model that uses the earnings process in Guvenen (2009) the insurance coefficients against permanent shocks are higher than in the baseline model. In the version with the tight borrowing constraint it is 0.33 and in the case with the natural borrowing constraint it is 0.4, even higher than the empirical target. This is consistent with the fact that if we constrain the whole profile of wealth over working life, shocks that have lower persistence will end up being better insured. The values of the elasticity of inter-temporal substitution that minimize the distance between model and data life-cycle wealth pattern is 0.8 and 0.7 respectively in the model with and without borrowing, the latter value being interior to the explored region. The risk aversion coefficients are 9 and 10 respectively and in the case of the model with the natural borrowing limit, the fraction of agents with negative wealth is 13.4 percent, close to the larger value reported in Huggett (1996).

Taken together the results in this section suggest that given the empirically observed pattern of wealth accumulation over working life, a standard incomplete market model is consistent with the estimates of insurance coefficients provided in Blundell *et al.* (2008): the model with the zero borrowing constraint can explain 83 percent of the consumption smoothing observed in the data, while the model with the natural borrowing limit can explain up to 97.3 percent. This is obtained with values of risk aversion and the elasticity of inter-temporal substitution that are in line with experimental and empirical evidence and, when debt is allowed, with a percentage of agents having negative wealth that is also consistent with the data.

5 Conclusions

In the current paper we have constructed a life-cycle version of the SIM model that is the workhorse of current heterogeneous agents macroeconomics. We have modified the standard version to adopt Epstein-Zin preferences and used it to explore its predictions with respect to the insurance coefficients proposed by Blundell *et al.* (2008). We have shown that the higher flexibility allowed by the preferences adopted here allows the model to bridge the gap between the insurance coefficients for permanent shocks produced by the model under standard expected utility, as described in Kaplan and Violante (2010), and the data. At the same time, the model is able to make the insurance coefficients flatter with respect to age although in this case the gap is not fully closed. This result is obtained with preference parameters that are within the limit of the empirical and experimental literature. Moreover, the associated life-cycle wealth profiles are more in line with the empirical ones than those generated by the baseline model. Indeed in the natural borrowing limit case over 97 percent of observed insurance can be explained with the parametrization of the model that best matches the profile of wealth accumulation over the working life. This suggests that the failure of the baseline model with standard expected utility to match the empirical insurance coefficient for permanent shocks mainly reflects its under-prediction of wealth accumulation early in the working-life.

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