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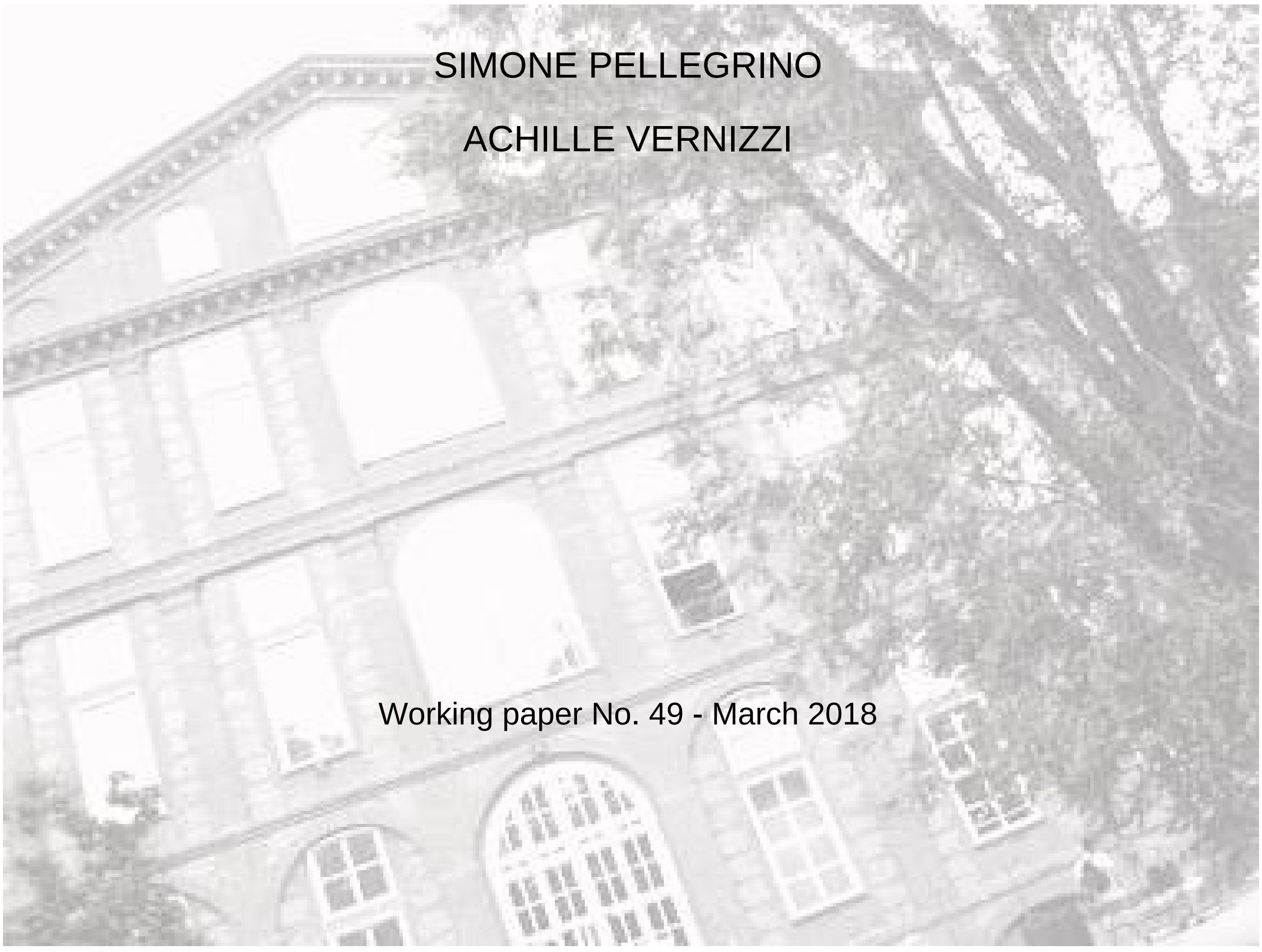

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Decomposing the Redistributive Effect of Taxation to Reveal Axiom Violations

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Abstract

In this paper we propose two alternative strategies in order to decompose the redistributive effect of the personal income tax in the portion due to deductions, marginal tax rates and tax credits. The first one, inspired by the analysis by Lambert [2001], Pfähler [1990] and Onrubia et al. [2014], is a stepwise or "ex ante" decomposition, whilst the second strategy, inspired by the works by Podder [1993a,b] and Podder and Chatterjee [2002], is an overall and simultaneous or "ex post" decomposition. The value added of our approaches is twofold: they are very simple and intuitive, and, moreover, both of them allow to quantify the Axiom violations, as proposed by Kakwani and Lambert [1998], for each part in which the redistributive effect can be decomposed. We take Italy as a case study.

JEL-Codes: C80, H23, H24.

Keywords : Personal income tax, Microsimulation models, Reynolds-Smolensky index, Pfähler decomposition, Kakwani and Lambert decomposition.

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1. Introduction

In his seminal paper, Pfähler [1990] proposes a sequential methodology to reveal the importance of marginal tax rates, deductions and allowances as well as tax credits in determining the overall redistributive effect of taxation, primary a personal income tax. Lambert [2001] refines this strategy focusing only on deductions as well as allowances and marginal tax rates, whilst recently Onrubia et al. [2014] revise and generalize these two redistributive effect decompositions.

These state-of-the-art approaches evaluate the effect of deductions by comparing the pre-tax distribution and the tax base one, the effect of the rate schedule by comparing the tax base distribution and the gross tax liability one, and, finally, the effect of tax credits by comparing the gross and net tax liability distributions.[1] In this respect, the first value added of our paper lies in the fact that we extend, focusing on a further perspective, different from the ones so far proposed in the literature, the range of the available decompositions by suggesting a new investigation on the interaction between the distribution of gross tax liability and the one observed for deductions and allowances. In so doing, we compare the gross tax liability distribution with the one that would be achieved were no deduction as well as allowances applied.

In addition, we observe that the Pfähler-Lambert-Onrubia procedures emphasize only a part of the "story", since they only focus on how the redistributive effect is achieved. Within this framework it is possible to explore which features of the tax contribute the most to the removal of a real-world tax from a perfectly equal one. In this paper we also study this issue, and we try to take a step further, by blending together the original idea of the redistributive effect decomposition by tax components to the Kakwani and Lambert [1998] approach for measuring inequity in taxation.

In particular, we propose two alternative and complementary methodologies to assess the importance of each element of the personal income tax in determining the overall redistributive effect.

The first methodology we consider evaluates a stepwise or "ex ante" redistributive effect of tax components; even if inspired by the original Pfähler-Lambert-Onrubia framework, our decomposition deviates from it, in order to proper engage the subsequent Kakwani-Lambert analysis. The mean goal is to measure the incremental effect of the redistributive effect due to deductions, deductions and rate schedule, and, finally, deductions and rate schedule as well as tax credits.

Our second methodology assesses an overall and simultaneous or "ex post" redistributive effect, and it is inspired by the works by Podder [1993a,b] and Podder and Chatterjee [2002]. The goal in this case is to measure the share of the redistributive effect due to each element of the tax, given the application of all the other tax elements.

Both the empirical strategies we devise are very simple and intuitive; moreover, in each part in which the redistributive effect can be decomposed, they let to quantify at which extent each element of the tax contributes to the Axiom violations that an equitable tax should respect. As a case study, we apply these new approaches to the Italian personal income tax, showing their potentialities.

The structure of the paper is as follows. Section 2 introduces the notation used throughout the paper. Section 3 summarises the existing literature on the redistributive effect of the personal income tax (Subsection 3.1) and its decomposition by tax components (Subsection 3.2), whilst Subsection 3.3 presents the three axioms proposed by Kakwani and Lambert [1998] when the whole tax structure is considered. Section 4 presents our methodologies. In particular, Subsection 4.1 focuses on the "step by

[1]For an exhaustive review and applications see Wagstaff and van Doorslaer [2001], Urban [2006] and Di Caro [2018].

step" analysis, whilst Subsection 4.2 describes the "overall and simultaneous" one. Both Subsections 4.1 and 4.2 are divided in two parts: the first one discusses our decomposition of the redistributive effect, whilst the second explains how Axiom violations can be evaluated for each part of the redistributive effect decomposition. Section 5 first presents the data and the microsimulation model employed in the empirical analysis (Subsection 5.1); it then shows results when considering the Italian personal income tax (Subsection 5.2). Section 6 offers a conclusion.

2. Starting Definitions

Considering a generic personal income tax, we start defining all the variables influencing the transition from the pre- to the post-tax income. A population of N income earners, with $i = (1, \dots, N)$, is considered. Let x_i be the taxpayer i's gross income. The corresponding taxable income b_i is given by the difference between the gross income x_i and all taxpayer i's allowances and deductions d_i. A rate schedule is applied to the taxable income b_i in order to derive the gross tax liability s_i. Finally, the net tax liability t_i is equal to the gross tax liability s_i minus all the tax credits c_i the taxpayer i can benefit.

We denote by $X = (x_1, \dots, x_N)$ the gross income distribution ordered in non decreasing order. Similarly, we call $D = (d_1, \dots, d_N)$ the distribution of tax allowances and deductions, $B = X - D = (b_1, \dots, b_N)$ the tax base distribution, $S = (s_1, \dots, s_N)$ the distribution of gross tax liabilities, $V = (v_1, \dots, v_N)$ the distribution of gross tax liabilities were the rate schedule applied to the X distribution instead of the B one; finally, we denote by $C = (c_1, \dots, c_N)$ the overall tax credit distribution, by $T = (t_1, \dots, t_N)$ the net tax liability distribution and by $Z = X - T = (z_1, \dots, z_N)$ the post-tax distribution.

Generically, we call $E = (\epsilon_1, \dots, \epsilon_N)$, where $E = (X, D, B, V, S, X-V, X-S, V-S, C, T, Z)$. To evaluate the inequality within these distributions, we employ the Gini [1914] coefficient $G_E = 2{\mu_E}^{-1}cov\big(E, F(E)\big)$ where μ_E is the average value of the considered distribution, cov represents the covariance, and $F(E)$ is the cumulative distribution function [Kakwani, 1980, Jenkins, 1988].

After the application of the tax, for each observed pair of values $\epsilon_i > \epsilon_j$, it is not granted that η_i and η_j are similarly ranked, where $H = (\eta_1, \dots, \eta_N)$ and $H = (X, D, B, V, S, X-V, X-S, V-S, C, T, Z)$. We then employ the concentration coefficient $C_{E|H} = 2{\mu_E}^{-1}cov\big(E, F(H)\big)$, where $C_{H|H} = G_H = G_E = C_{E|E}$ and $E|H$ represents the E distribution when its values are ranked according to the H order.

3. State of the Art

3.1. Basic Framework

Following the existing literature [Kakwani, 1977, Reynolds and Smolensky, 1977, Lambert, 2001], the overall redistributive effect is measured by $RE_T = G_X - G_Z = RS_T - R_{Z|X}$, where $RS_T = G_X - C_{Z|X}$ is the overall Reynolds-Smolensky index and $R_{Z|X} = G_Z - C_{Z|X}$ is the Atkinson-Plotnick-Kakwani index, which computes the extension of the re-ranking of post-tax incomes with respect to the pre-tax ones [Atkinson, 1980, Plotnick, 1981, Kakwani, 1984].

Similarly, the overall degree of tax progressivity is measured by the Kakwani index $K_T = C_{T|X} - G_X$.

As it is well known, RS_T and K_T are linked by the overall average tax rate $\theta = \frac{\sum_{i=1}^{N} t_i}{\sum_{i=1}^{N} x_i}$, since $RS_T = \frac{\mu_T}{\mu_Z} K_T$ and $\frac{\mu_T}{\mu_Z} = \frac{\theta}{1-\theta}$.

3.2. Decomposing the Reynolds-Smolensky Index by Tax Components

First of all the related literature focuses on decomposing the overall redistributive effect of the tax in three parts: the effect due to deductions and allowances, the one due to marginal tax rates, and the one due to tax credits. The seminal paper by Pfähler [1990] devises a first attempt [Onrubia et al., 2014]: it sequentially decomposes RE_T by focusing on the transition from distribution X to distribution B (deductions and allowances effect $RE_D = G_X - G_B$), from distribution B to distribution $B - S$ (rate schedule effect $RE_S = G_B - G_{B-S}$), and from distribution $B - S$ to distribution $B - T$ (tax credits effect $RE_C = G_{B-S} - G_{B-T}$):

$$RE_T = -\frac{\mu_T}{\mu_Z} RE_D + \frac{\mu_B - \mu_T}{\mu_Z} RE_S + \frac{\mu_B - \mu_T}{\mu_Z} RE_C. \tag{1}$$

Lambert [2001] revises this methodology by decomposing RS_T, and focusing only on deductions and allowances as well as marginal tax rates [Onrubia et al., 2014]:[2]

$$RS_T = -\frac{\mu_S}{\mu_{X-S}} \frac{\mu_{X-D}}{\mu_B} RS_D + \frac{\mu_{B-S}}{\mu_{X-S}} RS_S. \tag{2}$$

Finally, Onrubia et al. [2014] derive a generalization of Eq. 1 and 2:

$$RS_T = -\frac{S}{X-S} \sum_{o=1}^{O} \frac{\mu_{X-D_o}}{\mu_B} RS_{D_o} + \frac{\mu_{B-S}}{\mu_{X-S}} \sum_{l=1}^{L} \frac{\mu_{B-S_l}}{\mu_{B-S}} RS_{S_l} + \sum_{m=1}^{M} \frac{\mu_{X-S+C_m}}{\mu_Z} RS_{C_m} - R_{Z|X}. \tag{3}$$

When several deductions D_o and tax credits C_m as well as rate schedules S_l are considered [Kristjánsson, 2013], Eq. 3 allows to overcome the problem of sequentiality that characterizes the original framework, but it does not enable the decomposition of the overall re-ranking effect $R_{Z|X}$.

3.3. Considering Axiom Violations for the Whole Tax Structure

In their seminal paper, Kakwani and Lambert [1998] define three Axioms which should be respected by an equitable income tax. In particular, for each observed pair of values $x_i > x_j$, the first Axiom requires that $t_i > t_j$ (minimal progression), the second Axiom demands $\tilde{t_i} = \frac{t_i}{x_i} > \frac{t_j}{x_j} = \tilde{t_j}$, where $\tilde{t_i} = \frac{t_i}{x_i}$ is the taxpayer i's average tax rate (progressive principle), whilst Axiom 3 requests $z_i > z_j$ (no re-ranking).

By employing the re-ranking indexes of taxes $R_{T|X} = G_T - C_{T|X}$, tax rates $R_{\tilde{T}|X} = G_{\tilde{T}} - C_{\tilde{T}|X}$ (with $\tilde{T} = \frac{T}{X} = \tilde{t_1}, \ldots, \tilde{t_N}$), and post-tax incomes $R_{Z|X} = G_Z - C_{Z|X}$, the Authors show a measurement system[3] able to evaluate the negative influences that Axiom violations exert on the redistributive effect.[4]

More precisely, Kakwani and Lambert [1998] demonstrate that violations of Axiom 1 can be quantified as

$$AV_1^T = \frac{\mu_T}{\mu_Z} R_{T|X} \tag{4}$$

[2] See Urban [2006] for the extension of this methodology involving also tax credits.
[3] They state that Axiom 1 is violated if $R_{T|X} > 0$, Axiom 2 is violated if $R_{\tilde{T}|X} - R_{T|X} > 0$, and Axiom 3 if $R_{Y|X} > 0$.
[4] A revised empirical strategy can be found in Pellegrino and Vernizzi [2013].

whilst gross violations of Axiom 2 as[5]

$$AV_2^T = \frac{\mu_T}{\mu_Z} R_{\tilde{T}|X} \tag{5}$$

net violations of Axiom 2 as

$$AV_2^N = \frac{\mu_T}{\mu_Z}\left(R_{\tilde{T}|X} - R_{T|X}\right) = AV_2^T - AV_1^T \tag{6}$$

and violations of Axiom 3 as

$$AV_3^T = R_{Z|X}. \tag{7}$$

4. A New Approach

First of all we are interested in decomposing RS_T in three parts: the effect due to the rate schedule V (that is the transition from distribution X to distribution $X - V$), the effect due to deductions D (the transition from distribution V to distribution S, that also explains the transition from distribution X to distribution $X - S$), and the one due to tax credits C (the transition from distribution S to distribution T). Our goal is then to further decompose each of three portions in further three parts explaining the violations of the three Axioms the tax should respect.

As mentioned in the introduction, we first derive a "step by step" analysis, able to assess the stepwise redistributive effect and Axiom violations of each tax component, and then a proper "overall and simultaneous" decomposition of both the redistributive effect and Axiom violations.

4.1. The "Step by Step" Analysis

If no tax were applied, the redistributive effect would clearly be zero. Starting from this situation, it is useful to evaluate the redistributive effect that would be reached if only the rate schedule were applied. It is then interesting to compare the result with the redistributive effect achieved because of the rate schedule *and* deductions: the difference between these two values is due to the impact of deductions only.

Let consider a set of income earners, homogeneous with respect to a given equivalence scale; when applying a rate schedule with increasing marginal tax rates, deductions increase the redistributive effect that would be reached in their absence, if deductions are constant or (a fortiori) decreasing with respect to income. Conversely, if deductions are a direct function of income, they decrease the redistributive effect. A similar discussion can be reserved to tax credits, if the effect of the relative equivalence scale should be mimed by this tax instrument: constant (or decreasing) with respect to income tax credits increase the redistributive effect, whilst they decrease it, if they are a direct function of income.[6]

All this information set can be quantified by applying Equations interpreted in Subsection 4.1.

Subsection 4.1.2 focuses on Axiom violations, for each element of the tax discussed in Subsection 4.1.1.

[5]Differently from Kakwani and Lambert [1998], in what follows we consider the gross violations of Axiom 2, without considering if Axion 1 is either violated or not. For an interpretation of this methodology see Mazurek and Vernizzi [2013].

[6]We observe that tax fairness can request either deductions or tax credits to increase with income. As an example, consider a relative equivalence scale that is a direct function of the number of households components, and a personal income tax whose tax base is given by the nominal gross income minus deductions (and not the nominal gross income divided by the equivalence scale). In this case deductions should be increasing with income. An analogous consideration applies to tax credits if they should mimic the relative equivalence scale effect. If deductions (or tax credits) were constant (or decreasing with income), this would produce violations of the Kakwani and Lambert [1998] Axioms.

The structure of a real-world tax may be different (and specifically it is) from a perfectly equal one, primarily if the structure of each element of the tax is not consistent with the adopted equivalent scale: the incremental effect of Axiom violations due to rate schedule, rate schedule *and* deductions, and then tax credits is quantified.

4.1.1. The Decomposition of the Reynolds-Smolensky index

Were the rate schedule applied to the X distribution instead of the $B = X - D$ one, we would have

$$RS_V = G_X - C_{(X-V)|X} = \frac{\mu_V}{\mu_{X-V}}\Big(C_{V|X} - G_X\Big) = \frac{\mu_V}{\mu_{X-V}}K_V \tag{8}$$

that represents the effect due to the rate schedule. In real-world taxes, the rate schedule is instead applied to the tax base distribution $B = X - D$. As a consequence,

$$RS_S = G_X - C_{(X-S)|X} = \frac{\mu_S}{\mu_{X-S}}\Big(C_{S|X} - G_X\Big) = \frac{\mu_S}{\mu_{X-S}}K_S \tag{9}$$

depicts the effect exerted by both deductions and statutory marginal tax rates.[7]

The incremental effect due to deductions RS_D^{Δ} can be simply evaluated by $RS_S - RS_V$:

$$RS_D^{\Delta} = RS_S - RS_V = -\frac{\mu_{V-S}}{\mu_{X-S}}\Big(C_{(V-S)|X} - C_{(X-V)|X}\Big). \tag{10}$$

If $C_{(V-S)|X} - C_{(X-V)|X}$ is greater than zero, RS_D^{Δ} is negative, stemming for a detrimental contribution of deductions with respect to a tax system allowing only statutory marginal tax rates. Finally, the incremental effect due to tax credits RS_C^{Δ} can be similarly computed by

$$RS_C^{\Delta} = RS_T - RS_S = -\frac{\mu_C}{\mu_Z}\Big(C_{C|X} - C_{(X-S)|X}\Big) \tag{11}$$

that underlines that C determines a positive contribution to RS_T, with respect the same tax system not allowing tax credits, only if $C_{C|X} - C_{(X-S)|X}$ is lower than zero. Summarizing, we obtain:

$$RS_T = RS_V + RS_D^{\Delta} + RS_C^{\Delta} \tag{12}$$

with $RS_S = RS_V + RS_D^{\Delta}$.

4.1.2. The Issue of Axiom Violations

For each Axiom we show how its overall violations can be decomposed by the parts due to V, S, D and C.

Starting from Axiom 1, we state that

$$AV_1^T = AV_1^V + AV_1^D + AV_1^C. \tag{13}$$

In particular,

$$AV_1^V = \frac{\mu_V}{\mu_{X-V}}R_{V|X} \tag{14}$$

[7] In general, μ_V is expected to be greater than μ_S, and, consequently, μ_{X-V} is expected to be lower than μ_{X-S}, so that $\frac{\mu_V}{\mu_{X-V}} > \frac{\mu_S}{\mu_{X-S}}$.

$$AV_1^S = \frac{\mu_S}{\mu_{X-S}} R_{S|X} \tag{15}$$

$$AV_1^D = AV_1^S - AV_1^V \tag{16}$$

$$AV_1^C = AV_1^T - AV_1^S. \tag{17}$$

Having generically defined $\tilde{E} = \frac{E}{X}$ and $R_{\tilde{E}|X} = G_{\tilde{E}} - C_{\tilde{E}|X}$, we similarly evaluate the overall gross violations of Axiom 2 as follows:

$$AV_2^T = AV_2^V + AV_2^D + AV_2^C \tag{18}$$

where

$$AV_2^V = \frac{\mu_V}{\mu_{X-V}} R_{\tilde{V}|X} \tag{19}$$

$$AV_2^S = \frac{\mu_S}{\mu_{X-S}} R_{\tilde{S}|X} \tag{20}$$

$$AV_2^D = AV_2^S - AV_2^V \tag{21}$$

$$AV_2^C = AV_2^T - AV_2^S. \tag{22}$$

For what concerns[8] Axiom 3,

$$AV_3^T = G_Z - C_{Z|X} \tag{23}$$

where

$$AV_3^V = G_{X-V} - C_{(X-V)|X} \tag{24}$$

$$AV_3^S = G_{X-S} - C_{(X-S)|X} \tag{25}$$

$$AV_3^D = AV_3^S - AV_3^V \tag{26}$$

$$AV_3^C = AV_3^T - AV_3^S. \tag{27}$$

4.2. The "Overall and Simultaneous" Analysis

The decompositions introduced in Subsection 4.1 show how the "step by step" or "ex ante" application of tax instruments sequentially modifies the redistributive effect RS_T and influences Axiom violations.

As already observed, this strategy measures the incremental effect of the redistributive effect due to deductions, deductions *and* rate schedule, and, finally, deductions *and* rate schedule *as well as* tax credits (that is the whole tax structure).

[8] It is worth observing that a reduction of AV_1^T or AV_2^T, obtained by simply ordering T and $\tilde{T}$ according to X, respectively, would probably increase AV_3^T.

It is also interesting to consider another perspective: this is done in the present Section. Here we consider the "end" of the "story" and focus on the "overall and simultaneous" or "ex post" decompositions. They let to measure the share of the redistributive effect due to each element of the tax, *given* the application of *all the other* tax elements. For example, researchers could be interested in apprehending the effect of the rate schedule on the redistributive effect, knowing that deductions and tax credits have been already considered within the analysis.

In this Section we also show at which extent the elements of the tax contribute to Axiom violations.

Even if the magnitudes and the signs of the two versions of the redistributive effect decomposition could be expected to be similar, nothing can be "a priori" said on the composition of Axiom violations, as we will discuss in greater details in Subsections 5.2.2 and 5.2.3.

4.2.1. The Decomposition of the Reynolds-Smolensky index

RS_T can be decomposed in three "overall and simultaneous" components, each of them able to isolate the effect due to V, S, D, and C:

$$RS_T = \frac{\mu_T}{\mu_Z} K_T = RS_V^* + RS_D^* + RS_C^* \tag{28}$$

where

$$K_T = \frac{\mu_V}{\mu_T}\Big(C_{V|X} - G_X\Big) - \frac{\mu_{V-S}}{\mu_T}\Big(C_{(V-S)|X} - G_X\Big) - \frac{\mu_C}{\mu_T}\Big(C_{C|X} - G_X\Big). \tag{29}$$

Moreover, following Podder [1993a,b] and Podder and Chatterjee [2002], it is interesting to note that Eq. 28 can be reconsidered by employing the Gini correlation coefficient $r_{E|H} = \frac{C_{E|H}}{G_E}$, which evaluates the rank of E with respect to the rank of H:

$$RS_T = \frac{\mu_T}{\mu_Z}\left(\frac{\mu_V}{\mu_T}\Big(r_{V|X}G_V - G_X\Big) - \frac{\mu_{V-S}}{\mu_T}\Big(r_{(V-S)|X}G_{V-S} - G_X\Big) - \frac{\mu_C}{\mu_T}\Big(r_{C|X}G_C - G_X\Big)\right) \tag{30}$$

where

$$RS_V^* = \frac{\mu_V}{\mu_Z}\Big(r_{V|X}G_V - G_X\Big) \tag{31}$$

$$RS_S^* = RS_V^* + RS_D^* \tag{32}$$

$$RS_D^* = -\frac{\mu_{V-S}}{\mu_Z}\Big(r_{(V-S)|X}G_{V-S} - G_X\Big) \tag{33}$$

$$RS_C^* = -\frac{\mu_C}{\mu_Z}\Big(r_{C|X}G_C - G_X\Big). \tag{34}$$

On one hand Eq. 30 informs on the Gini coefficients of V, $V-S$ and C; on the other it informs on the intensity and the direction of the correlation between V, $V-S$ and C with respect to X.

4.2.2. The Issue of Axiom Violations

For what concerns the first Axiom, focusing on Eq. 28, 29 and 30, the following equation holds:

$$\begin{aligned} AV_1^T &= \frac{\mu_T}{\mu_Z} R_{T|X} = AAV_1^V + AAV_1^D + AAV_1^C = AAV_1^S + AAV_1^C = \\ = \frac{\mu_T}{\mu_Z}\Bigg(\frac{\mu_V}{\mu_T}\Big(r_{V|T} - r_{V|X}\Big)G_V &- \frac{\mu_{V-S}}{\mu_T}\Big(r_{(V-S)|T} - r_{(V-S)|X}\Big)G_{V-S} - \frac{\mu_C}{\mu_T}\Big(r_{C|T} - r_{C|X}\Big)G_C\Bigg). \end{aligned} \tag{35}$$

In particular,

$$AAV_1^V = \frac{\mu_V}{\mu_Z}(r_{V|T} - r_{V|X})G_V \tag{36}$$

$$AAV_1^S = AAV_1^V + AAV_1^D \tag{37}$$

$$AAV_1^D = -\frac{\mu_{V-S}}{\mu_Z}\Big(r_{(V-S)|T} - r_{(V-S)|X}G_{V-S}\Big) \tag{38}$$

$$AAV_1^C = -\frac{\mu_C}{\mu_Z}\Big(r_{C|T} - r_{C|X}G_C\Big) = AV_1^T - AAV_1^S. \tag{39}$$

Continuing with the second Axiom,

$$\begin{aligned} AV_2^T &= \frac{\mu_T}{\mu_Z} R_{\tilde{T}|X} = AAV_2^V + AAV_2^D + AAV_2^C = AAV_2^S + AAV_2^C = \\ = \frac{\mu_T}{\mu_Z}\Bigg(\frac{\mu_{\tilde{V}}}{\mu_{\tilde{T}}}\Big(r_{\tilde{V}|\tilde{T}} - r_{\tilde{V}|X}\Big)G_{\tilde{V}} &- \frac{\mu_{\widetilde{V-S}}}{\mu_{\tilde{T}}}\Big(r_{(\widetilde{V-S})|\tilde{T}} - r_{(\widetilde{V-S})|X}\Big)G_{\widetilde{V-S}} - \frac{\mu_{\tilde{C}}}{\mu_{\tilde{T}}}\Big(r_{\tilde{C}|\tilde{T}} - r_{\tilde{C}|X}\Big)G_{\tilde{C}}\Bigg). \end{aligned} \tag{40}$$

In particular,

$$AAV_2^V = \frac{\mu_T}{\mu_Z}\frac{\mu_{\tilde{V}}}{\mu_{\tilde{T}}}\Big(r_{\tilde{V}|\tilde{T}} - r_{\tilde{V}|X}\Big)G_{\tilde{V}} \tag{41}$$

$$AAV_2^S = AAV_2^V + AAV_2^D \tag{42}$$

$$AAV_2^D = -\frac{\mu_T}{\mu_Z}\frac{\mu_{\widetilde{V-S}}}{\mu_{\tilde{T}}}\Big(r_{(\widetilde{V-S})|\tilde{T}} - r_{(\widetilde{V-S})|X}\Big)G_{\widetilde{V-S}} \tag{43}$$

$$AAV_2^C = -\frac{\mu_T}{\mu_Z}\frac{\mu_{\tilde{C}}}{\mu_{\tilde{T}}}\Big(r_{\tilde{C}|\tilde{T}} - r_{\tilde{C}|X}\Big)G_{\tilde{C}}. \tag{44}$$

Finally, focusing on the last Axiom, we start considering the decompositions of G_Z and C_Z:

$$G_Z = \frac{\mu_{(X-V)}}{\mu_Z}C_{(X-V)|Z} + \frac{\mu_{(V-S)}}{\mu_Z}C_{(V-S)|Z} + \frac{\mu_C}{\mu_Z}C_{C|Z} \tag{45}$$

$$C_Z = \frac{\mu_{(X-V)}}{\mu_Z}C_{(X-V)|X} + \frac{\mu_{(V-S)}}{\mu_Z}C_{(V-S)|X} + \frac{\mu_C}{\mu_Z}C_{C|X} \tag{46}$$

and, evaluating their difference, AV_3^T follows:

$$AV_3^T = \frac{\mu_{X-V}}{\mu_Z}\Big(r_{(X-V)|Z} - r_{(X-V)|X}\Big)G_{X-V} + \\ \frac{\mu_{V-S}}{\mu_Z}\Big(r_{(V-S)|Z} - r_{(V-S)|X}\Big)G_{V-S} + \frac{\mu_C}{\mu_Z}\Big(r_{C|Z} - r_{C|X}\Big)G_C. \tag{47}$$

In particular,

$$AAV_3^V = \frac{\mu_{X-V}}{\mu_Z}\Big(r_{(X-V)|Z} - r_{(X-V)|X}\Big)G_{X-V} \tag{48}$$

$$AAV_3^S = AAV_3^V + AAV_3^D \tag{49}$$

$$AAV_3^D = \frac{\mu_{V-S}}{\mu_Z}\Big(r_{(V-S)|Z} - r_{(V-S)|X}\Big)G_{V-S} \tag{50}$$

$$AAV_3^C = \frac{\mu_C}{\mu_Z}\Big(r_{C|Z} - r_{C|X}\Big)G_C. \tag{51}$$

5. The Empirical Analysis

5.1. The Data

As input data we make use of a static microsimulation model developed by Pellegrino [2007] about 10 years ago and constantly updated. It is able to estimate the most important taxes and contributions which characterize the Italian fiscal system. Here we employ the microsimulation model module concerning the personal income tax updated to the 2014 fiscal year.[9]

The microsimulation model employs, as input data, those provided by the Bank of Italy [2015] in its Survey on Household Income and Wealth (BI-SHIW), published in 2015 with regard to the 2014 fiscal year. The BI-SHIW survey contains information on household income and wealth of 8,156 households and 19,366 individuals. The sample is representative of the Italian population, composed of about 24.7 million households and 60.8 million individuals.

Considering individual taxpayers, results concerning the gross income distribution, and the distribution of all tax variables as well as the overall tax revenue are very close to the Department of Finance [2016] official statistics. Moreover, inequality indexes both for taxpayers and equivalent households are also very similar to the ones evaluated by the Department of Finance official microsimulation model [Di Nicola et al., 2015]. The instrument employed in this paper is then suitable for the type of empirical analysis we propose.

Starting from taxpayers distributions, we derive the corresponding equivalent households ones by applying the equivalent scale given by the square root of the number of the components. In the following Subsection results are presented and discussed according to these equivalent households distributions.

[9]Technical details regarding the structure and main results of this version of the microsimulation model can be found in Pellegrino et al. [2017], whilst previous applications of the model to the analysis of the Italian personal income tax and the analysis of inequality indexes decompositions can be view in Pellegrino et al. [2011], and Monti et al. [2015], respectively.

5.2. Results

5.2.1. Basic Indexes

Table 1 shows all the basic as well as composed inequality indexes involved in our decompositions; it also presents the average values of the tax variables. The Gini coefficient for the pre-tax distribution G_X is equal to 0.42089, whilst the concentration coefficient for the post-tax one $C_{Z|X}$ is 0.37035. As a consequence, the overall Reynolds-Smolensky index RS_T is equal to 0.05054. This is the value we would like to decompose by isolating the effect of the most important tax components. The Gini coefficient for the post-tax distribution G_Z equals 0.37097, so that the overall redistributive effect RE_T is 0.04992, which is lower than RS_T because of the re-ranking occurred to net incomes in the transition from the pre- to the post-tax values (measured by Axiom 3, which is the Atkinson-Plotnick-Kakwani index $R_{Z|X} = G_Z - C_Z = AV_3^T = 0.00062$). The concentration coefficient for net tax liabilities $C_{T|X}$ is 0.63954, and the Kakwani index $K_T = C_{T|X} - G_X$ is equal to 0.21865, $\frac{\mu_T}{\mu_Z} = \frac{\theta}{1-\theta}$ to 0.23113 and θ to 0.18774. As observed for net incomes, also G_T=0.64626 is greater than $C_{T|X}$; their difference $R_{T|X} = 0.00673$ evaluates the importance of the re-ranking occurred to the net tax liability distribution because of the tax.[10]

The Gini coefficient for tax credits is $G_C = 0.22783$, whilst the corresponding concentration coefficients are very low: $C_{C|X} = 0.04587$ and $C_{C|T} = 0.02299$. Differently, the Gini coefficient evaluated for deductions is very high ($G_D = 0.69886$), whilst the corresponding concentration coefficient is remarkably lower ($C_{D|X} = 0.48105$). These indexes basically say that deductions are enjoyed by a not very high share of income units, and these income units are distributed along the all X distribution, with a more crowded concentration on middle and high incomes; conversely, tax credits are enjoyed by the vast majority of income units, and they are distributed along all the pre-tax distribution.

As shown in Subsections 4.1.2 and 4.2.2, deductions exert an effect in the transition from the V distribution to the S one, whilst tax credits from the S distribution to the T one. We will focus on the impact of the related indexes later; here we start noticing their values. For what concerns the two gross tax liability distributions, $G_V = 0.48343$, whilst $G_S = 0.47901$; the corresponding concentration coefficients are $C_{V|X} = 0.48245$, $C_{V|T} = 0.47992$, $C_{S|X} = 0.47732$ and $C_{S|T} = 0.47595$. Moreover, $G_{V-S} = 0.74211$, $G_{X-V} = 0.39786$ and $G_{X-S} = 0.40147$, whilst $C_{(V-S)|X} = 0.56811$, $C_{(X-V)|X} = 0.39768$ and $C_{(X-S)|X} = 0.40123$. Finally, discussing basic indexes specifically employed for Axiom 2, $G_{\tilde{T}} = 0.42087$, $C_{\tilde{T}|X} = 0.39600$, $G_{\tilde{S}} = 0.08561$, $C_{\tilde{S}|X} = 0.06919$, $C_{\tilde{S}|\tilde{T}} = 0.07325$, $G_{\tilde{V}} = 0.07271$, $C_{\tilde{V}|X} = 0.06686$, $C_{\tilde{V}|\tilde{T}} = 0.06930$, $G_{V\tilde{-}S} = 0.64961$, $C_{(V\tilde{-}S)|X} = 0.02633$, $C_{(V\tilde{-}S)|\tilde{T}} = 0.00096$, $G_{\tilde{C}} = 0.35939$, $C_{\tilde{C}|X} = -0.28435$, $C_{\tilde{C}|\tilde{T}} = -0.30281$.

All the inequality decompositions we propose depend on the average values of the distributions they refer to: the average gross income μ_X is 21,615.47 euros, whilst $\mu_V = 5,918.43$, $\mu_S = 5,583.88$, $\mu_{V-S} = 334.56$, $\mu_{X-V} = 15,697.04$, $\mu_{X-S} = 16,031.60$, $\mu_C = 1,525.82$, $\mu_T = 4,058.06$, μ_Z is 17,557.41, $\mu_{\tilde{V}} = 0.24437$, $\mu_{\tilde{S}} = 0.23104$, $\mu_{V\tilde{-}S} = 0.01333$, $\mu_{\tilde{C}} = 0.11098$ and $\mu_{\tilde{T}} = 0.12006$.

We can now start analysing the core structure of our methodologies. First we present the "step by step" analyses (Subsection 5.2.2), and then the "overall and simultaneous" decompositions (Subsection 5.2.3).

[10]Were the T distribution ordered exactly like the X one, G_T would be equal to $C_{T|X}$. This is not the case in real-world situations, since the structure of the personal income tax is very complex, characterized by dozens of parameters [Morini and Pellegrino, 2018], each of them influencing the D and C orderings with respect the X one, and in turn the V, S and T ones.

Table 1: Inequality indexes values

Index	Value
G_X	0.42089
G_D	0.69886
$C_{D\|X}$	0.48105
G_V	0.48343
$C_{V\|X}$	0.48245
$C_{V\|T}$	0.47992
$C_{V\|Z}$	0.48042
G_S	0.47901
$C_{S\|X}$	0.47732
$C_{S\|T}$	0.47595
$C_{S\|Z}$	0.47481
G_{V-S}	0.74211
$C_{(V-S)\|X}$	0.56811
$C_{(V-S)\|T}$	0.54614
$C_{(V-S)\|Z}$	0.57401
G_{X-V}	0.39786
$C_{(X-V)\|X}$	0.39768
$C_{(X-V)\|T}$	0.39173
$C_{(X-V)\|Z}$	0.39746
G_{X-S}	0.40147
$C_{(X-S)\|X}$	0.40123
$C_{(X-S)\|T}$	0.39495
$C_{(X-S)\|Z}$	0.40115
G_C	0.22783
$C_{C\|X}$	0.04587
$C_{C\|T}$	0.02299
$C_{C\|Z}$	0.05388
G_T	0.64626
$C_{T\|X}$	0.63954
G_Z	0.37097
$C_{Z\|X}$	0.37035
$G_{\tilde{V}}$	0.07271
$C_{\tilde{V}\|X}$	0.06686
$C_{\tilde{V}\|\tilde{T}}$	0.06930
$G_{\tilde{S}}$	0.08561
$C_{\tilde{S}\|X}$	0.06919
$C_{\tilde{S}\|\tilde{T}}$	0.07325
$G_{V\tilde{-}S}$	0.64961
$C_{V\tilde{-}S\|X}$	0.02633
$C_{V\tilde{-}S\|\tilde{T}}$	0.00096
$G_{\tilde{C}}$	0.35939
$C_{\tilde{C}\|X}$	-0.28435
$C_{\tilde{C}\|\tilde{T}}$	-0.30281
$G_{\tilde{T}}$	0.42087
$C_{\tilde{T}\|X}$	0.39600

Source: Own elaborations.

Index	Value
RS_T	0.05054
RE_T	0.04992
K_T	0.21865
AV_1^T	0.00155
AV_2^N	0.00419
AV_2^T	0.00575
$AV_3^T = R_{Z\|X}$	0.00062
$R_{T\|X}$	0.00673
$R_{V\|X}$	0.00098
$R_{S\|X}$	0.00169
$R_{\tilde{V}\|X}$	0.00586
$R_{\tilde{S}\|X}$	0.01642
$R_{\tilde{T}\|X}$	0.02486
$r_{V\|X}$	0.99797
$r_{V\|T}$	0.99274
$r_{(V-S)\|X}$	0.76554
$r_{(V-S)\|T}$	0.73593
$r_{(V-S)\|Z}$	0.77348
$r_{(X-V)\|X}$	0.99953
$r_{(X-V)\|Z}$	0.99899
$r_{C\|X}$	0.20135
$r_{C\|T}$	0.10093
$r_{C\|Z}$	0.23649
$r_{\tilde{V}\|X}$	0.91947
$r_{\tilde{V}\|\tilde{T}}$	0.95312
$r_{(V\tilde{-}S)\|X}$	0.04053
$r_{(V\tilde{-}S)\|\tilde{T}}$	0.00148
$r_{\tilde{C}\|X}$	-0.79120
$r_{\tilde{C}\|\tilde{T}}$	-0.84258
θ	0.18774
$\frac{\mu_T}{\mu_Z}$	0.23113
μ_X	21,615.47
μ_V	5,918.43
μ_S	5,583.88
μ_{X-V}	15,697.04
μ_{X-S}	16,031.60
μ_{V-S}	334.56
μ_C	1,525.82
μ_T	4,058.06
μ_Z	17,557.41
$\mu_{\tilde{V}}$	0.24437
$\mu_{\tilde{S}}$	0.23104
$\mu_{V\tilde{-}S}$	0.01333
$\mu_{\tilde{C}}$	0.11098
$\mu_{\tilde{T}}$	0.12006

5.2.2. The "Step by Step" Analysis

If the tax structure considered only statutory tax rates, the overall redistributive effect would be $RS_V = 0.02321$. Considering this situation, and adding deductions and allowances, the redistributive effect is lower ($RS_S = 0.01965$). This means that the marginal effect of D is detrimental for the overall redistributive effect, since $RS_D^\Delta = RS_S - RS_V = -0.00356$. Finally, if we also add tax credits C, we obtain the final RS_T, remarkable greater than RS_S; this means that the marginal effect of C, $RS_C^\Delta = 0.03088$, contributes the most to RS_T (Table 2).

Table 2: RS_T analysis – Step by step

	RS_V	RS_D^Δ	RS_S	RS_C^Δ	RS_T
Value	0.02321	−0.00356	0.01965	0.03088	0.05054

Source: Own elaborations.

Focusing on Axiom 1, Table 3 shows the marginal effects of V, D, S and C in determining AV_1^T. In particular, AV_1^V is equal to 0.00037, lower than $AV_1^S = 0.00059$. If we compare the situation with and without deductions, it follows that D positively contributes to increase Axiom 1 violations. A similar picture emerges if tax credits C are added in the analysis: $AV_1^T = 0.00155$ is greater than $AV_1^S = 0.00059$, stemming for a positive effect of C in increasing Axiom 1 violations ($AV_1^C = 0.00097$).

Table 3: Violations of Axiom 1 – Step by step

	AV_1^V	AV_1^D	AV_1^S	AV_1^C	AV_1^T
Value	0.00037	0.00022	0.00059	0.00097	0.00155

Source: Own elaborations.

Table 4: Violations of Axiom 2 – Step by step

	AV_2^V	AV_2^D	AV_2^S	AV_2^C	AV_2^T
Value	0.00221	0.00351	0.00572	0.00003	0.00575

Source: Own elaborations.

Table 5: Violations of Axiom 3 – Step by step

	AV_3^V	AV_3^D	AV_3^S	AV_3^C	AV_3^T
Value	0.00019	0.00005	0.00024	0.00038	0.00062

Source: Own elaborations.

Table 4 shows the corresponding marginal effects of V, D, S and C in determining AV_2^T. In particular, AV_2^V is equal to 0.00221, remarkably lower than $AV_2^S = 0.00572$. This is due to the positive contribute of D in increasing AV_2^T, since AV_2^D is positive and equal to 0.00351. Again a similar picture emerges if tax credits C are considered: $AV_2^T = 0.00575$ is only a little greater than $AV_2^S = 0.00572$, stemming for a positive but negligible effect of tax credits C in increasing this Axiom violations ($AV_2^C = 0.00003$). Finally, AV_3^V is equal to 0.00019, lower than $AV_1^S = 0.00024$; $AV_3^T = 0.00062$ is greater than AV_1^S.

As a consequence, again both D and C positively contribute to Axiom 3 violations: $AV_3^D = 0.00005$ and $AV_3^C = 0.00038$ (Table 5).

5.2.3. The "Overall and Simultaneous" Analysis

First of all, the RS_T decomposition can be observed in Table 6, which presents actual values and values as a percentage of RS_T.

Table 6: RS_T decomposition – Overall

	RS_V^*	RS_D^*	RS_S^*	RS_C^*	RS_T
Value	0.02075	−0.00281	0.01795	0.03259	0.05054
% of RS_T	41.06	−5.55	35.51	64.49	100.00

Source: Own elaborations.

Given the application of deductions and tax credits, the share of RS_T due to the marginal tax rates applied to the X distribution ($RS_V^* = 0.02075$) is 41.06%, whilst the corresponding share due to the marginal tax rates applied to the S distribution ($RS_S^* = 0.01795$) is lower, 35.51%. This means that the application of deductions reduces by 5.55% the potential effect due to marginal tax rates ($RS_D^* = -0.00281$). Finally, given the application of deductions and marginal tax rates, the presence of tax credits explains 64.49% of the overall redistributive effect ($RS_C^* = 0.03259$).

As briefly observed before, Table 2 and Table 6 present the same signs and similar actual values. We can also notice some specific relations for the three couples of indexes. For example, both Eq. 8 and Eq. 31 (which evaluate RS_V and RS_V^*, respectively) consider the same rate schedule progressivity term $\left(C_{V|X} - G_X\right) = \left(r_{V|X}G_V - G_X\right)$, but Eq. 8 multiplies it by $\frac{\mu_V}{\mu_{X-V}}$, whilst Eq. 31 by $\frac{\mu_V}{\mu_Z}$. Since, generally speaking, $\mu_Z > \mu_{X-V}$, RS_V is expected to be greater than RS_V^*. For what concerns Eq. 10 and Eq. 33, since $\mu_Z > \mu_{X-S}$, it follows that $RS_D^\Delta < RS_D^*$ if $C_{(X-V)|X} < G_X$. Finally, Eq. 11 differs from Eq. 34 depending on the values of $C_{(X-S)|X}$ and G_X. Since, as expected, the latter is greater than the former, then $RS_C^\Delta < RS_C^*$.

The compositions depicted in Table 2 and Table 6 are confirmed if the methodologies proposed by Pfähler [1990] and Onrubia et al. [2014] are applied to the Italian income distribution [Barbetta et al., 2018]; as a consequence, without twisting the original framework and its results, we can sieve all the effects of the tax components by focusing on each Axiom violations, on which we turn from now on.

Table 7 presents the decomposition of Axiom 1, whilst Table 8 and Table 9 those of Axiom 2 and 3, respectively. These tables show actual values and values as a percentage of RS_T as well as AV_1^T, AV_2^T and AV_3^T, respectively.

More precisely, AV_1^T is 3.08% of RS_T (Table 7, second row). Deductions D and tax credits C positively contribute to Axiom 1 violations by 0.83% (AAV_1^D) and 3.93% (AAV_1^C), respectively: their effects are detrimental for AV_1^T, and they are offset by the rate schedule (−1.69% if AAV_1^V is considered and −0.86% if AV_1^S is instead contemplated). Regarding the composition of AV_1^T (Table 7, third row), tax credits C contribute the most to the overall violation: AAV_1^C represents 127.90% of AV_1^T, whilst AAV_1^D is 26.93%. Together they reach 154.83%, so that the rate schedule (AAV_1^V) overcomes these unpleasant outcomes by −54.83%.

AV_2^T is 11.37% of RS_T (Table Table 8, second row). All elements of the tax positively contribute to Axiom 2 violations: V by 2.28% of RS_T (AAV_2^V), D by 1.29% (AAV_2^D) and C by 7.81% (AAV_2^C).

Focusing on the composition of AV_2^T (Table 8, third row), tax credits C contribute the most to the overall violation: AAV_2^C represents 68.64% of AV_2^T, AAV_2^S 31.35%, AAV_2^V 20.03% and AAV_2^D 11.32%.

For what concerns the third Axiom, its overall violations are a low percentage (1.22%) of RS_T (Table 9, second row). As observed for Axioms 1, deductions D and tax credits C positively contribute also to this Axiom violations: $AAV_3^D = 0.22\%$ and $AAV_3^C = 1.38\%$. On the contrary, the rate schedule counterpoises these effects: $AAV_3^V = -0.38\%$ and $AAV_3^S = -0.16\%$. This is particularly noticeable by looking to the composition of AV_3^T (Table 9, third row): tax credits contribute for 112.88% to the overall effect, whilst deductions only for 18.23%; these percentages are reduced by the negative contribute of the marginal tax rates (-31.10% and -12.87%, respectively).

Table 7: Violations of Axiom 1 – Overall

	AAV_1^V	AAV_1^D	AAV_1^S	AAV_1^C	AV_1^T
Value	−0.00085	0.00042	−0.00043	0.00199	0.00155
% of RS_T	−1.69	0.83	−0.86	3.93	3.08
% of AV_1^T	−54.83	26.93	−27.90	127.90	100.00

Source: Own elaborations.

Table 8: Violations of Axiom 2 – Overall

	AAV_2^V	AAV_2^D	AAV_2^S	AAV_2^C	AV_2^T
Value	0.00115	0.00065	0.00180	0.00394	0.00575
% of RS_T	2.28	1.29	3.57	7.81	11.37
% AV_2^T	20.03	11.32	31.35	68.64	100.00

Source: Own elaborations.

Table 9: Violations of Axiom 3 – Overall

	AAV_3^V	AAV_3^D	AAV_3^S	AAV_3^C	AV_3^T
Value	-0.00019	0.00011	-0.00008	0.00070	0.00062
% of RS_T	-0.38	0.22	-0.16	1.38	1.22
% AV_3^T	-31.10	18.22	-12.88	112.88	100.00

Source: Own elaborations.

5.2.4. Comparing the Two Methodologies

By comparing Tables 3, 4 and 5 with Tables 7, 8 and 9 it can be noted that Axiom violations measures are considerably different if the "ex ante" or the "ex post" approach are considered: not only the contribution of each component can be different, but its sign can change, due to the sign of the differences between the Gini correlation coefficients in the corresponding Equations.

Focusing on Axiom 1 and Axiom 3, the "ex post" analysis shows even negative signs for the rate schedule contributions V and S to the corresponding overall Axiom violations. In particular, their negative effect partially compensates the positive effect due to deductions D and tax credits C. Also the magnitude of the positive contributions due to D and C are very different between the two approaches; in particular, the "ex ante" contributions are about a half than the "ex post" ones.

On the contrary, focusing on Axiom 2, the "ex post" contribution due to D is remarkably lower (about one fifth) than the corresponding "ex ante" one. The opposite happens if tax credits C are examined: in particular, the "ex ante" methodology shows that the marginal contribution of tax credits is not only very low, but also the lowest among all the tax components, whilst the situation is reversed if the "ex post" methodology is considered: in this case it is the contribution of C to be remarkably high.

An intuition on both the sign and the magnitude of these effects can be derived by interpreting Eq. 14-17, Eq. 19-22, Eq. 24-27, and, similarly, Eq. 36-39, Eq. 41-44, Eq. 48-51 according to the Pellegrino and Vernizzi [2013] framework, able to isolate the contribution due to every possible pair-wise comparison in determining the Gini and concentration coefficients.[11]

For example, focusing on Axiom 1, and without going into too much details (since they are beyond the scope of present paper), the following relationships can be observed.

For what concerns the "ex ante" approach, and looking at the contribution of a pair of income units (i, j), if $(v_i - v_j)$ has a sign not opposite to $(x_i - x_j)$ the effect on AV_1^V is zero, so that no Axiom violation happens; otherwise $((v_i - v_j)$ has a different sign with respect to $(x_i - x_j))$, the contribution on AV_1^V is positive. A similar discussion can be reserved to the other two steps: instead of $(v_i - v_j)$, in these cases $(s_i - s_j)$ and $(t_i - t_j)$ have to be replaced. At each step, a pair of values (i, j) can add a further violation, diminish the violation of the previous step, or annihilate the intensity of violation registered in the previous step.

In the "ex post" approach, a pair of income units (i, j) is not considered at all whenever it does not contribute to the final violation of Axiom 1; this happens when $x_i > x_j$ $(x_i < x_j)$, so that $t_i \geq t_j$ $(t_i \leq t_j)$; for $x_i = x_j$ both $t_i \geq t_j$ and $t_i \leq t_j$ are allowed. Eq. 35 considers only the pairs of values which present a reciprocal ranking in the distribution T ordering, opposite to the one they have within distribution X; this means that Eq. 35 includes only income pairs which contribute to the final violation of Axiom 1. In particular, if $(v_i - v_j)$ and $(x_i - x_j)$ as well as $(t_i - t_j)$ have all the same sign, their effect on AAV_1^V is zero; their effect is also zero if $(t_i - t_j)$ and $(x_i - x_j)$ have the same sign, opposite to the one observed for $(v_i - v_j)$. Finally, if $(t_i - t_j)$ has a different sign with respect to both $(x_i - x_j)$ and $(v_i - v_j)$ (that is t_i and t_j violate the first Axiom), the contribution on AAV_1^V is negative; if both $(t_i - t_j)$ and $(v_i - v_j)$ have the same sign, opposite to the one observed for $(x_i - x_j)$, their contribution is positive instead.

A similar discussion holds for deductions D (Eq. 38) and tax credits C (Eq. 39); in these circumstances Axiom 1 is violated if distributions $S - V$ and C oppose distribution X and follow distribution T.

In general, having considered all possible pair-wise comparisons which violate AV_1^T, the more the V and S distributions ensue the X one, the higher are the negative contributions of V and S in explaining

[11] For a generic attribute E the Gini and concentration coefficients can be evaluated as follows:

$$G_E = \frac{1}{2\mu_E N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} (e_i - e_j) I_{i-j}^{E}$$

$$C_{E|X} = \frac{1}{2\mu_E N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} (e_i - e_j) I_{i-j}^{E|X}$$

where

$$I_{i-j}^{E} = \begin{cases} +1 & \text{if } e_i \geq e_j \\ -1 & \text{if } e_i < e_j \end{cases}$$

and

$$I_{i-j}^{E|X} = \begin{cases} +1 & \text{if } x_i > x_j \\ I_{i-j}^{E} & \text{if } x_i = x_j \\ -1 & \text{if } x_i < x_j. \end{cases}$$

Axiom 1 violations; similarly, the more the V and S distributions ensue the T one, the higher are the positive contributions of V and S in explaining Axiom 1 violations. A similar reasoning can be derived for deductions and tax credits: the more the D and C distributions ensue the X (T) one, the higher are the positive (negative) contributions of D and C in explaining Axiom 1 violations.

Mutatis mutandis, a close discourse (here omitted) can be done for Axiom 2 and Axiom 3.

6. Concluding Remarks

Considering each fundamental element of the personal income tax, the aim of this paper is to reveal the importance of Axiom violations, as introduced by Kakwani and Lambert [1998], in each part the redistributive effect can be decomposed.

As it is well known, Pfähler [1990], Lambert [2001] and Onrubia et al. [2014] propose methodologies able to divide the overall redistributive effect in the portion due to deductions, marginal tax rates and tax credits. Kakwani and Lambert [1998] instead advance an axiomatic approach to measure the Axioms violations an equitable tax should respect.

We link together these two branches of the literature in order not only to minutely explain how the redistributive effect is achieved, but also to expose the causes determining the removal of a real-world personal income tax from a perfectly equal one.

To reach this goal, we propose two new decompositions of the redistributive effect compatible with the axiomatic approach by Kakwani and Lambert [1998]. The value added of our strategy is its mathematical simplicity, which makes it more understandable also to researchers not familiar with these fundamental analyses of the personal income tax.

More precisely, we also divide the redistributive effect in three parts: the effect due to the rate schedule, deductions, and tax credits. Our decompositions present some peculiarities not involved in previous attempts. Moreover, each of these three portions are further decomposed in further three parts explaining the violations of the three Axioms the tax should respect.

We apply this methodology to the Italian personal income tax, showing the capillary analysis that is possible to infer with these instruments. They can be particularly useful for policy makers when they think of a tax reform aimed at better choose the tax parameters in order to increase its progressivity and in the meanwhile to reduce the regressiveness implicitly existing in imperfect real-world taxes.

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