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A NOTE ON THE MAXIMUM VALUE OF THE KAKWANI INDEX

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A Note on the Maximum Value of the Kakwani Index

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Abstract

The overall tax revenue of a real world personal income tax cannot be eventually paid only by the richest taxpayer. Therefore, the maximum concentration coefficient for taxes cannot be equal to 1, and, consequently, the maximum value of the Kakwani index cannot be 1 minus the Gini coefficient for pre-tax incomes, as generally described in the related literature. We give evidence of this phenomenon by illustrating a theoretical example, and by evaluating its maximum value when a real world tax is considered.

JEL-Codes: H23, H24.
Keywords : Kakwani index, Redistributive effect, Personal income tax, Microsimulation models.

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1. Introduction

In their seminal papers, Jakobsson [1976], Fellman [1976], Kakwani [1977] and Reynolds and Smolensky [1977] show how the degree of progression and the redistributive effect of a tax can be quantified. In particular, Kakwani [1977] proposes his famous index able to compute the departure from proportionality of a progressive income tax. This index measures the difference between the concentration coefficient for the tax liability distribution and the the Gini coefficient for the pre-tax income one.

All the related tax literature (e.g., Lambert [2001]) states that its maximum value is one minus the Gini coefficient for the pre-tax income distribution, and its minimum value -1 minus the same Gini coefficient. We argue that these phenomena can happen in one special case that is not satisfied in real world personal income taxes. As a consequence, the maximum (minimum) value of the Kakwani index is lower (greater) than its theoretical one. Focusing on the maximum value, we give evidence of its magnitude by an example regarding the Italian income taxation.

The structure of the paper is as follows. Section 2 first presents the basic inequality indexes (Subsection 2.1) and then focuses on the highest admittable value of the Kakwani index (Subsection 2.2); finally, Subsection 2.3 discusses a stylized example. Section 3 briefly introduces the data and the microsimulation model employed in this work, and subsequently reports the results. Section 4 concludes.

2. Inequality Indexes

2.1. Basic Notation

A population of N income earners, with $i = 1, \ldots, N$, is considered. We denote by $X = (x_1, \ldots, x_N)$ the gross income distribution ordered in non decreasing order. Similarly, we call $T = (t_1, \ldots, t_N)$ the tax liability distribution and $Z = (z_1, \ldots, z_N)$ the post-tax income one.

To evaluate the inequality within these distributions, we employ the Gini [1914] coefficient $G_\epsilon = 2{\mu_\epsilon}^{-1} cov\big(\epsilon, F(\epsilon)\big)$ and the corresponding concentration one $C_{\epsilon|\eta} = 2{\mu_\epsilon}^{-1} cov\big(\epsilon, F(\eta)\big)$, where $\epsilon, \eta = (X, T, Z)$, $C_{\eta|\eta} = G_\eta = G_\epsilon = C_{\epsilon|\epsilon}$, μ_ϵ is the average value of the considered distribution, cov represents the covariance, and $F(\epsilon)$ is the cumulative distribution function [Kakwani, 1980, Jenkins, 1988]. As it is well known, Gini and concentration coefficients range between zero and $\frac{N-1}{N}$, $1 = \lim_{N\to\infty} \frac{N-1}{N}$ in case of large samples.

Following the existing literature [Lambert, 2001], the redistributive effect RE can be measured by $RE = G_X - G_Z = RS - RR$ where $RS = G_X - C_{Z|X}$ is the Reynolds-Smolensky index and $RR = G_Z - C_{Z|X}$ is the Atkinson-Plotnick-Kakwani index. Similarly, the degree of tax progressivity can be computed by the Kakwani index $K = C_{T|X} - G_X$, linked to RS by the overall average tax rate $\theta = \frac{\sum_{i=1}^N t_i}{\sum_{i=1}^N x_i}$: $RS = \frac{\theta}{1-\theta} K$.

2.2. The Maximum Value

For large samples, all the tax literature states that the maximum value of the Kakwani index is $K^{MAX} = 1 - G_X$ and its minimum value is $K^{MIN} = -1 - G_X$.

These extreme bounds are possible under the condition that the highest admittable value for $C_{T|X}$ is 1 and the corresponding minimum value is equal to -1.

It has to be noted that the above mentioned extreme values for K can be verified in one special case: the overall tax revenue is lower than the top (bottom) gross income x_N (x_1) observed in the income distribution.

This is not what researchers observe in real world taxation. If this restrictive hypothesis is relaxed, the highest value of the tax liability concentration $C_{T|X}^{MAX}$ is necessarily lower than 1 and the corresponding lowest value $C_{T|X}^{MIN}$ is greater than -1. As a consequence, K^{MAX} (K^{MIN}) depends on the distribution of X and the overall amount of the tax revenue to be collected.

2.3. A Simple Example

Suppose a uniform distribution of $N = 10$ pre-tax values ranging from 10 to 100 with jumps of 10 monetary units: $X = (10, 20, \ldots, 90, 100)$. In this case $G_X = 0.3$.

Suppose initially that only the richest taxpayer has to face a positive tax liability. Until the tax revenue Υ is lower than or at most equal $x_{10} - x_9 = 10$, $C_{T|X} = \frac{N-1}{N} = 0.9$, and the maximum Kakwani index is $K = \frac{N-1}{N} - G_X = 0.6$. In addition, $G_T = C_{T|X}$, and $RR = 0$.

If $x_{10} - x_9 = 10 < \Upsilon \leq 100 = x_{10}$, the maximum K is still 0.6, but $RR > 0$ and it monotonically increases with Υ. For all possible values $\Upsilon > 100 = x_{10}$, two or more taxpayers are needed for Υ to be paid,[1] so that $K < \frac{N-1}{N} - G_X$.

Figure 1 shows the maximum K as a function of Υ: it is constant and equal to $\frac{N-1}{N} - G_X$ until $\Upsilon \leq x_{10}$, but subsequently it monotonically decreases and reaches zero when $\Upsilon = 550$.

To understand this relation, Table 1 provides distributions T and Z for three specific tax revenue amounts Υ^j (with $j = 1, 2, 3$): $\Upsilon^1 = x_{10}$, $\Upsilon^2 = x_{10} + x_9$ and $\Upsilon^3 = x_{10} + x_9 + \delta$, where $\delta < x_8$; Table 2 illustrates the inequality indexes.

As mentioned above, if $\Upsilon^1 = x_{10} = 100$ the richest taxpayer gets a net income $z_{10}^1 = 0$, so that K^1 is equal to 0.6, but $RR^1 = 0.2$ and $RE^1 < 0$ even if $RS^1 > 0$.

If $\Upsilon^2 = x_{10} + x_9 = 190$ the two richest taxpayers get $z_{10}^2 = z_9^2 = 0$, and they pay $t_{10}^2 = 100$ and $t_9^2 = 90$ of taxes, respectively. In this circumstance $C_{T|X}^2$ cannot be equal to its theoretical maximum value (0.9), but it is lower (0.80526), so that also K^2 is lower (0.50526 instead of 0.6). Note that RR^2 undergoes a severe deterioration with respect to RR^1, even if $RS^2 > RS^1$.

If $\Upsilon^3 = x_{10} + x_9 + \delta = 250$ the two richest taxpayers get $z_{10}^3 = z_9^3 = 0$, whilst $z_8^3 = 20$, and $t_8^3 = 60$. As can be noted, K^3 is lower than K^2, and RR^3 continues its deterioration. And so on.

3. An Application to a Real World Tax

We make use of a static microsimulation model concerning the Italian personal income tax [Pellegrino, 2007] updated to the 2014 fiscal year [Pellegrino et al., 2017].[2]

As input data, it employs those provided by the Bank of Italy [2015] in its Survey on Household Income and Wealth,[3] published in 2016 with regard to the 2014 fiscal year.

According to the microsimulation model, the 2014 overall tax revenue Υ is 151.7 billion euros. Having ranked pre-tax values in non decreasing order and considered sample weights, the top 1.5 million taxpayers (3.7% of all) earn a pre-tax income equal to Υ. Supposing all these taxpayers face a tax liability equal to their income, and the remaining ones a zero tax liability, $C_{T|X}^{MAX} = 0.97371 < 1$. Since $G_X = 0.45253$, it follows that $K^{MAX} = 0.52118$ instead of 0.54747. Moreover, note that RR^{MAX} is 0.07311, whilst the observed RR according to the actual tax code is 0.00088, 83 times lower.

[1] In the example under discussion the highest admittable value for the tax revenue is $\Upsilon = \sum_{i=1}^{10} x_i = 550$, that occurs when each taxpayer pays a tax liability equal to his gross income.

[2] Results of the model are very close to the Department of Finance [2016] official statistics. Moreover, inequality indexes both for taxpayers and equivalent households are also very close to the ones evaluated by the Department of Finance official microsimulation model [Di Nicola et al., 2015].

[3] The survey contains information on income and wealth of 8,156 households and 19,366 individuals, and it is representative of the Italian population, composed of about 24.7 million households and 60.8 million individuals.

Figure 1: The maximum Kakwani index as a function of the tax revenue

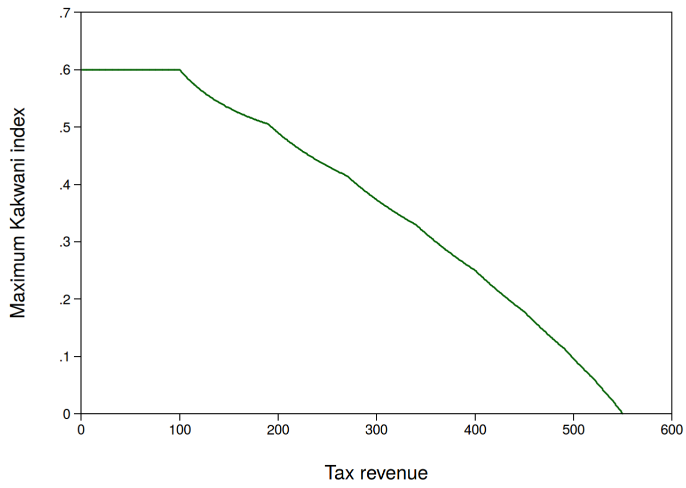

Table 1: Specific cases

i	x_i	t_i^1	z_i^1	t_i^2	z_i^2	t_i^3	z_i^3
1	10	0	10	0	10	0	10
2	20	0	20	0	20	0	20
3	30	0	30	0	30	0	30
4	40	0	40	0	40	0	40
5	50	0	50	0	50	0	50
6	60	0	60	0	60	0	60
7	70	0	70	0	70	0	70
8	80	0	80	0	80	60	20
9	90	0	90	90	0	90	0
10	100	100	0	100	0	100	0

Source: Own elaborations.

Table 2: Inequality indexes

	$\Upsilon^1 = 100$	$\Upsilon^2 = 190$	$\Upsilon^3 = 250$
θ	0.18182	0.34545	0.45455
G_X	0.30000	0.30000	0.30000
$C_{T\|X}$	0.90000	0.80526	0.73200
K	0.60000	0.50526	0.43200
G_Z	0.36667	0.43333	0.44000
$C_{Z\|X}$	0.16667	0.03333	-0.06000
RR	0.20000	0.40000	0.50000
RS	0.13333	0.26667	0.36000
RE	−0.06667	−0.13333	−0.14000

Source: Own elaborations.

4. Concluding Remarks

In this paper we stress that, even if desired, the overall tax revenue of a personal income tax cannot be concentrated only on the richest (poorest) income earner, simply because the overall tax revenue is remarkable greater than the top (bottom) gross income observed in real world income distributions.

From this simple observation follows that the maximum concentration coefficient for taxes cannot be 1, and, consequently, the maximum value of the Kakwani index cannot be equal to 1 minus the Gini coefficient for pre-tax incomes as generally described in the related literature.

We give evidence of this phenomenon by illustrating a theoretical example; we then evaluate its maximum value by considering a real world tax.

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