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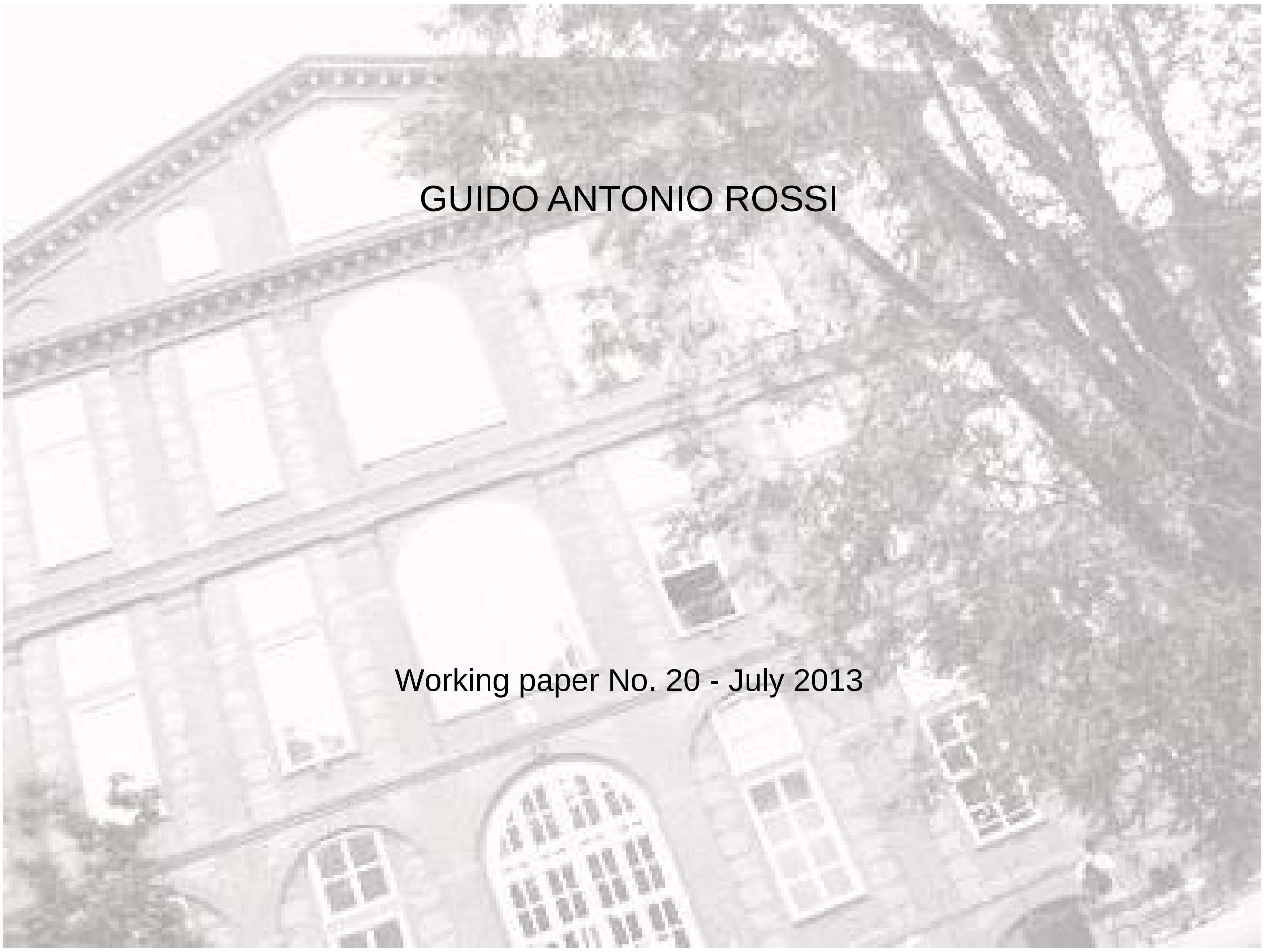
ON THE PROBABILITY THAT NOTHING HAPPENS



GUIDO ANTONIO ROSSI

Working paper No. 20 - July 2013

On the Probability that Nothing Happens*

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Abstract

We analyse the probability that an isolated system remain unchanged along time, finding its general expression showing that it can have discontinuities incompatible with the countable additivity principle**.
JEL Classification: C1, keyword *Probabilities.* Mathematical Subject Classification(2010): 60Axx, 60Gxx.

0 - **Introduction**

Probability that nothing happens is to be considered as following:

-let us imagine an isolated system, existing at least from sometime onwards and subject to evaluation during time flow;

-we consider the possibility of the system remaining totally unchanged or instead being subject to any possible change, some trace of which will in any case remain making it definitive, even if apparently the change might be reversed (as an example demolishing a house and then rebuilding it will not give us exactly the old one as curators of heritage buildings know very well).

We can then consider a stochastic process having two states only: 0, the initial one, and 1, the other one which is absorbing.

We shall study with no restrictive hypothesis the transition probability from state 0 in some instant x to the same state 0 in a following instant y. Its complement to 1 is the probability that something happens between x and y.

1 - **Stochastic process**

1.1 Given a stochastic process depending on time with two states 0 initial and 1 absorbing, let P(x, y) be the transition probability from state 0 at epoch x to state 0 at epoch y, with $y \geq x$; the Chapman-Kolmogorov equation gives the following functional equation to be solved:

(1) $P(x, y) = P(x, z)\ P(z, y)$, subject to $x < z < y$ when $y \neq x$,

while $P(x, x)$ - the probability that no change of state happen in the very given instant x, complement to one of the probability that such a change happen - is not subject to such an equation but must be analysed apart according to the case.

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*The present paper deals with a purely theoretical topic and a well constrained one, however extensions are foreseen and important applications together with empirical analyses also.
**A partial presentation of the content of an earlier version of this paper, which lacked what is in section 4 and following, was publicly appreciated by Bruno de Finetti.

1.2 Should we extend equation (1) to y =x,it would be necessary to put P(x,x) =1, which means that it would be almost impossible that something happen in a single instant. The same conclusion would be reached by assuming that P (x,y) be partially continuous, assumption that we do not make. Partial continuity would descend from the countable additivity principle, but we do not adopt it either. Should we impose P(x,x) =1 we would obtain a function identical to the probability that nothing happens conditional on the fact that no change happens exactly at x.

1.3 We shall solve equation (1) in section 4, giving a necessary and sufficient condition under no restrictive hypothesis; we shall study properties of P, in section 2 the preliminary ones and in in section 5 those coming from the solution.
We shall develop some use of the solution in some detail in section 6: the most outstanding case of such a probability is perhaps the survival probability of a single individual subject a fixed set of possible death causes, where 0 is being alive and 1 is being dead (its complement to 1 is the probability of dying between x and y).

2- **First properties of P**

2.0 Let us examine immediate properties.

2.0.1 The domain of P(x,y) is the superior semi plane with respect to the bisector of the axes in the plane x,y, where P(x,y) is governed by equation (1) above the bisector, and a separate study is to be done on the frontier line y=x. A graphic representation (see fig.1) will be helpful:

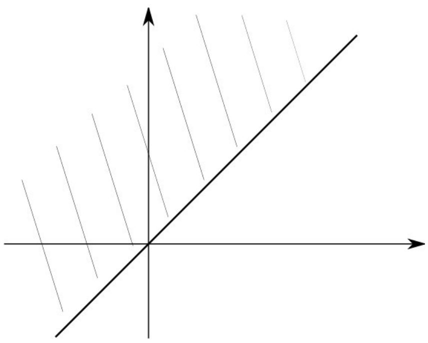

Fig. 1

The combination law of (1) regards points (see fig.2) as represented:

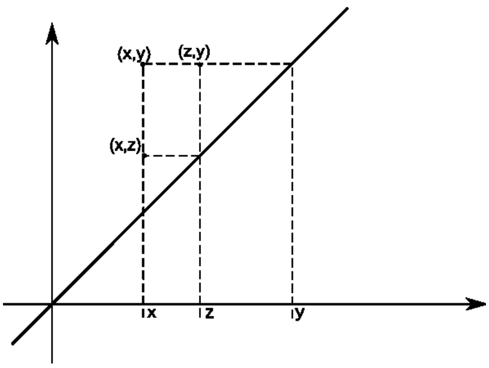

Fig. 2

2.0.2 It is obvious that (1) has two peculiar solutions:

$P(x,y) \equiv 0$ and $P(x,y) \equiv 1$, which are lower and superior bounds for all solutions, and mean that it is almost impossible either to stay in state 0 or to arrive in state 1.

2.0.3 We cannot suppose P to be always positive then; to go on we shall study the following properties of solutions of (1) : monotonicity, sets of zeroes, continuity.

2.1. Monotonicity

2.1.1 Let us first analyse monotonic behaviour along vertical and horizontal lines, for $h, k > 0$:

D (x,y+h)=P(x,y) P(y,y+h)$\leq P(x,y)$because $P(y, y+h) \leq 1$, and similarly

P(x-h,y) =P(x-h,x)P(x,y) $\leq P(x,y)$, and then

P(x-h,y+k)$\leq P(x,y)$.

2.1.2 So $P(x,y)$is weakly increasing along all straight lines horizontal, vertical or of negative inclination when approaching the bisector and weakly decreasing when moving away; more generally it is weakly decreasing together with the length of interval [a,b], that is increasing with respect to x and decreasing with respect to y.

2.1.3 As a consequence on the bisector P cannot be smaller than its least upper bound in the neighbourhood of (x,x).

2.1.4.1 Given any (a,b), a <b, and the closed and unlimited quadrant $\{(x,y) \mid (x \leq a) \wedge (y \geq b)\}$, there it will be $P(x,y) \leq P(a,b)$ (see fig.3).

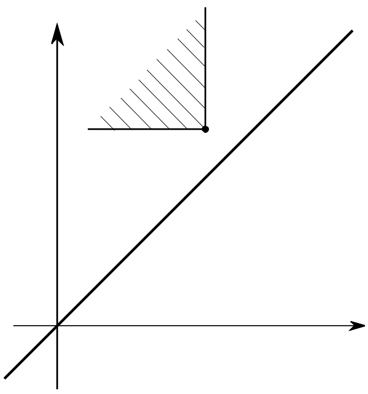

Fig. 3

2.1.4.2 Vice versa given any (a,b),$a < b$ and the intersection of the quadrant $\{(x,y) | (x \geq a) \wedge (y \leq b)\}$ with x≤y, there it will be P (x,y) ≥P(a,b) (see fig.4).

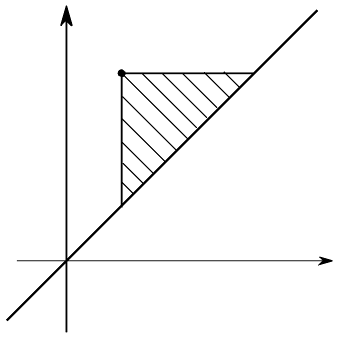

Fig. 4

2.1.5 We cannot get general monotonicity properties of P along directions of positive inclination from (a.b).

2.2 Sets of zeroes

2.2.1 If P(a,b) =0 then P (x,y) =0 for x≤a and y≥b, moreover from 0=P(a,b) =P(a,z)P(z, b) (with a <z<b), we obtain

at least one of the three $\begin{cases} P(a,z) = 0 \\ P(z,b) = 0 \\ P(a,z) = P(z,b) = 0 \end{cases}$, then either on the horizontal or on the vertical straight line from (a,b) to the bisector excluded, P is null. The set of zeroes originated by (a,b) is then a quadrant congruent to the second one and having the vertex on the bisector, closed on one side, but not necessarily on the other one or on the vertex. An easy corollary is that if (c,c) is the vertex with $a < c < b$ and P(a,b) =0, then $P(c,b) > 0$ implies $P(a,c) = 0$ or vice versa (see fig. 5).

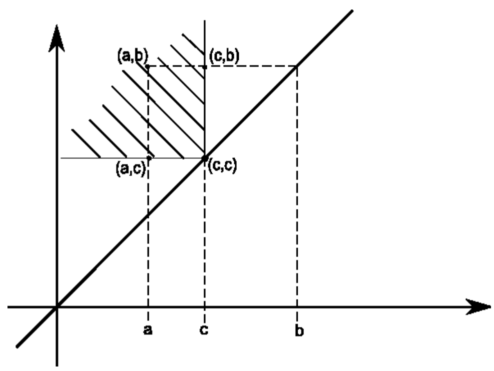

Fig. 5

2.2.2.1 Other sets of zeroes may result from a limit of sets of the previous type (see fig.6) being then open on both sides or even having vertices on a full interval (see fig.7), where it is sufficient that the supposed vertices actually proved to be so, be dense to obtain that the whole interval is of such vertices. On the frontier we can say nothing general. as

2.2.2.2 Taking as an example the survival of a single individual born at 0, we see that P is null for negative x and for y ≥ ω, ω the extreme age (see also 5.0.1)

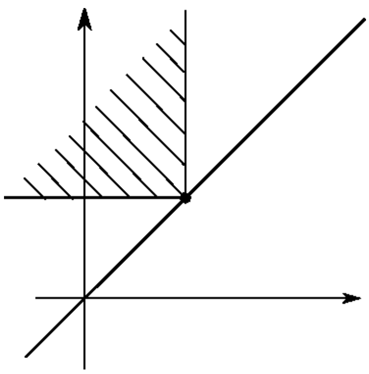

Fig. 6

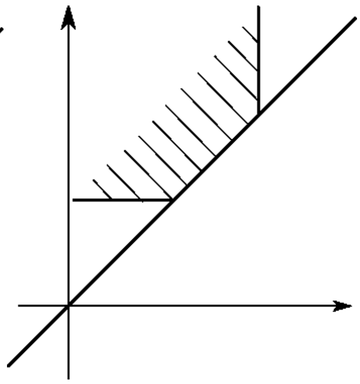

Fig. 7

2.2.3 Passing to the complementary set where P is positive we see that it is one or more triangles, eventually unlimited, having one side horizontal, one vertical and the third on the bisector (see fig.8): they can be described by an inequality $a < x < y < b$ each, with possible weakening of the extreme inequalities to be discussed case by case.

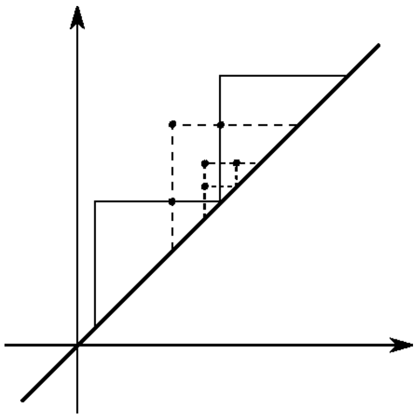

Fig. 8

2.3 Continuity

2.3.1 Introducing some restrictive hypotheses about continuity - the fact that it is imposed makes it clear that there might be something else - we shall derive from equation (1) some properties of P which will be later examined easily and confirmed in section 5, after having solved equation (1). We do that just in order to show that the properties involved are general and hold on the whole semi plane $y \geq x$ with a peculiar position on the bisector $y = x$.

2.3.2 Let us suppose that at a certain point (a,b) the function P(x,y) is positive and continuous from above with respect to y: $\lim_{h\to 0^+} P(a, b+h) = P(a,b) \neq 0.$

2.3.3 Then $\lim_{h\to 0^+} P(b, b+h) = 1$, because

$$P(a,b) = \lim_{h\to 0^+}\big(P(a,b)P(b,b+h)\big) = P(a,b)\lim_{h\to 0^+} P(b,b+h).$$

2.3.4 And then $\lim_{h\to 0^+} P(x, b+h) = P(x,b)$ for every $x < b$, *because*

$$\lim_{h\to 0^+} P(x,b+h) = \lim_{h\to 0^+}\big(P(x,b)P(b,b+h)\big) = P(x,b).$$

2.3.5 And if we suppose continuity from the left with respect to x:

$$\lim_{h\to 0^+} P(a-h,b) = P(a,b) \neq 0.$$

2.3.6 Then

$$\lim_{h\to 0^+} P(a-h,a) = 1, \text{because}$$
$$P(a,b) = \lim_{h\to 0^+}\big(P(a-h,a)P(a,b)\big) = \lim_{h\to 0^+} P(a-h,a)\; P(a,b).$$

2.3.7 And $\lim_{h\to 0^+} P(a-h,y) = P(a,y)$ for every $y > a$, *because*

$$\lim_{h\to 0^+} P(a-h,y) = \lim_{h\to 0^+}\big(P(a-h,a)P(a,y)\big) = P(a,y).$$

2.3.8 Obviously requiring an absolute continuity property we can address equation (1) through a partial differential equation in the sense of Carathéodory, as it was done in Rossi G.A.(1979b): it is not the case of doing that now because it is surpassed by what shall be found in section 5.

3- **Sincov functional equation**

3.1 Equation (1) recalls Sincov functional equation in multiplicative form and suggest the same solution, however the original multiplicative Sincov functional equation requires P always positive on the whole plane and no restriction on the variables as can be well described in the excellent survey paper Gronau D.(2000). We shall then need to confine ourselves to the set where P is positive modifying the original proof where needed, recalling that when P is null the equation is automatically solved.

4 **-Solving equation (1)**

4.1 Let us rewrite the original equation (1):

(2) $P(x,y) = P(x,z)P(z,y), x < z < y$ with $0 < P \leq 1$, having as domain the complement of the sets of zeroes above the bisector x=y, $a < x < z < y < b$, with eventually $a = -\infty$ and or $b = +\infty$. That is we solve equation (1) in any one of the triangles where P can be positive.

To solve the functional equation we can proceed as follows.

4.2 Since $P > 0$ we can transform (2) by means of a logarithm into
(3) $G(x,y) = G(x,z) + G(z,y)$ with $G(x,y), G(x,z), G(z,y) \leq 0$ and then proceed as with the Sincov functional equation though we have not an unconstrained universal quantifier for x, z, y but three different points in a given relationship are to be considered (see fig.9):

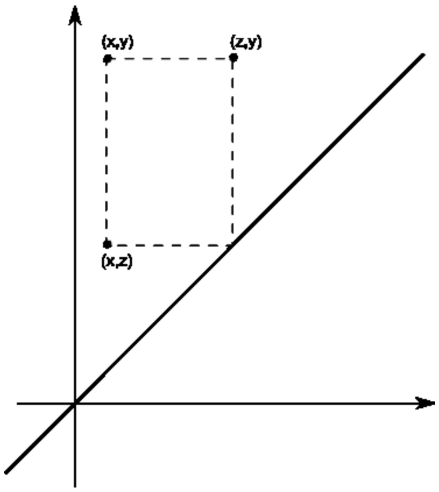

Fig. 9

4.3 Proof part I. We pass to look for a necessary condition by transforming (2) considered as true into (4) $G(z, y) = G(x, y) - G(x, z)$, obtaining that the variable x is not present on the left hand side of the equality and then G(z,y) does not depend upon it; and on the right hand side we have the same function determined at y and at z where x enters into determining the functional operator: we shall call G (x,t) as g(t), obtaining
(5) $G(z, y) = g(y) - g(z)$ with $g(y) \leq g(z)$ at point (z, y).

4.4 Remark. At this point with no constraints $G(x, y)$ and $G(x, z)$ are determined using the identity of operator G which we require to be the same on the whole domain by substantially switching the variables, so obtaining $G(x, y) = g(y) - g(x)$ at point (x, y) and $G(x, z) = g(z) - g(x)$ at point (x, z). However we feel that the conclusion would now be cursive, because the variables are not completely interchangeable as they are constrained, and it could be
$G(x, y) = g(y) - g(x) + H(x, y)$ and $G(x, z) = g(z) - g(x) + H(x, z)$ or
$G(x, y) = \big(g(y) - g(x)\big)K(x, y)$ and $G(x, z) = \big(g(z) - g(x)\big)K(x, z)$,
with $H(z, y) = 0$ and $K(z, y) = 1$ but not necessarily elsewhere.

4.5 Proof part II. So we can split the reasoning into two parts, and first show that at points (x,y) and (x,z), G has necessarily an additive form and then use this form to finally prove the result.

Substituting (5) into (4) we have:

$g(y) - g(z) = G(x,y) - G(x,z)$, which becomes

(6) $g(y) - G(x,y) = g(z) - G(x,z)$, and we see that on both sides we have the same function $g(t) - G(x,t)$ evaluated at different values of variable t=y and t=z; moreover the equality says that the function is constant with respect to this variable, only subject to $a < t < b$, and then the only true variable is x.

We can therefore write (6) as (7) h(x)=h(x), and then

(8) $\begin{cases} g(y) - G(x, y) = h(x) \\ g(z) - G(x, z) = h(x) \end{cases}$ or $\begin{cases} G(x, y) = g(y) - h(x) \\ G(x, z) = g(z) - h(x) \end{cases}$, obtaining again the additive form.

Then we can call h(x) as g(x) and operator G is always the same giving the general necessary condition G(x,y)=g(y) − g(x) with g(y) ≤ g(x) which is obviously sufficient for (3).

4.6 Conclusion. Finally it is necessary and sufficient for (2) and (1) that $P(x,y)=e^{g(y)-g(x)}$ on every set where P(x,y) > 0, with y > x; and when P(x,x) = 1 we can extend this form of P to y ≥ x.
In addition we have g(y) −g(x)≤ 0 to satisfy P(x, y)≤ 1 and then g(x) is to be weakly decreasing then continuous save for downward jumps.
And if g is constant on the interval h < t < k -and recalling the monotonicity of 2.1- P(x,y) = 1 for h < x ≤ y < k (see fig.4), with possible weakening of one or both of the inequalities relative to h or k and then closure on the catheti.

5-Control of general properties

5.0.1 Let us consider the fact that the system under observation may be (in fact is) not eternal and let us call α the moment the system starts to exist and is in its initial state 0 (for our universe it would be the big-bang), or the moment we start to observe it supposed existing: then 0 ≤P(α,α) ≤1 because a system might exist and remain in the original state for a single instant (so as we might consider survival probability of a born-dead infant) and if P(α,α)= 0 it is almost certain that it does not preserve its initial state beyond time α .

If P(x,y) >0 for a<x<y<b (b eventually +∞) then a ≥ α (see fig. 10)..

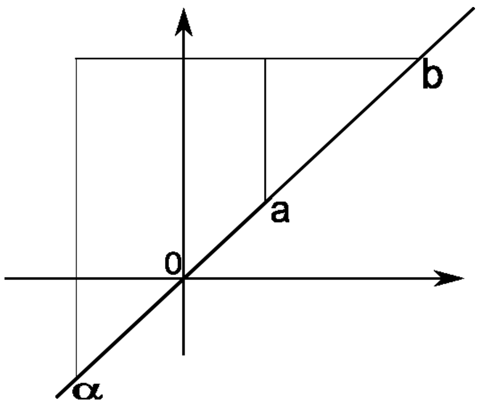

Fig. 10

5.0.2 Function g(t) is defined and unique up to any additive constant: if g(t) gives a definite P(x,y) also g(t) +c, for whichever constant c, gives the same P(x,y). We can then fix the value of g at any point, for example suppose that g(α) =1 or g(0) =1, and then g(a) ≤ 1.

5.1.1 Let us analyse continuity of P using g which we suppose continuous in t_0 :
if $\lim_{h\to 0^+} g(t_0 + h) = g(t_0)$
$P(x,y)=e^{g(y)-g(x)}$ is continuous on the right on $x=t_0$ for any $y> t_0$ and from above on $y=t_0$ for any $x< t_0$;

if $\lim_{h\to 0^+} g(t_0 - h) = g(t_0)$

$P(x,y)=e^{g(y)-g(x)}$ is continuous on the left on $x=t_0$ for any $y> t_0$ and from below on $y=t_0$ for any $x< t_0$.

In addition

$\lim_{h\to 0^+} e^{g(t_0+h)-g(t_0)} = \lim_{h\to 0^+} e^{g(t_0)-g(t_0-h)} =1$ and then $P(t_0, t_0)= 1$.

5.1.2 We see easily that continuity of a certain type at a single point in the domain where P is positive provokes characteristic continuities along entire vertical or horizontal straight lines on the same domain, properties that are immediately extended on the set of zeroes, but behaviour on the frontiers is not general and depends on the single case.

5.1.3 Let us assume that $P(x,y)> 0$ for $a<x<y<b$ $(a \geq \alpha)$.

5.1.3.1 Then if $\lim_{y\to+\infty} P(x,y) = 0$ without being constant from some value of y onwards, then

$\lim_{y\to+\infty} g(y) = -\infty$ as $0= e^{-g(x)} \lim_{y\to+\infty} e^{g(y)}$.

5.1.3.2 Instead if P(x,y) is null for any x if y>b and P(x,y) is continuous in y=b then $\lim_{y\to b^-} g(y) = -\infty$ and P(x,b) =0.Discontinuity of P(x,y) at y=b can b on the left or on the right or both. Since P(x,y) is null for any y if $x<\alpha$ (eventually $\alpha=0$) P(x,y) is surely not continuous at $x = y =\alpha$ from the left.

5.2 Function g(t) is generally decreasing and then subject to the decomposition theorem of Lebesgue, thus $g(t)= g_1(t)+g_2(t) + g_3(t)$ where g_1 is absolutely continuous, g_2 is a step function and g_3 is the singular component; so $P(x,y) = e^{g_1(y)-g_1(x)} \cdot e^{g_2(y)-g_2(x)} \cdot e^{g_3(y)-g_3(x)}$ and each component of g is to be decreasing by itself.
Note that in order to receive no discomfort from g_3 it is enough that it be constant, not necessarily null.

5.3.1 Let us study the case of g with a discontinuity at t_0, $a < t_0 < b$: a downward jump of amplitude h (>0).
Ignoring g_3 but not g_1 which is continuous, if $a<x<y< t_0$, $g(y)= g_1(y) +h$, $g(x)= g_1(x) +h$, and if $t_0 <x<y<b$, $g(y)= g_1(y)$, $g(x)= g_1(x)$, then on the bisector P(t,t) =1 for $t\neq t_0$ because of continuity of g_1 (see fig.11).

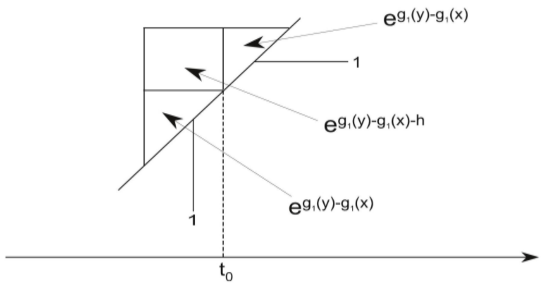

Fig. 11

5.3.2 We have to look with care at what happens on the bisector.

5.3.2.1 At t_0 we have the cases:

(a) $g(t) = h + g_1(t)$ for $t \le t_0$, $g(t) = g_1(t)$ for $t > t_0$

(b) $g(t) = h + g_1(t)$ for $t < t_0$, $g(t) = g_1(t)$ for $t \ge t_0$

(c) $g(t) = h + g_1(t)$ for $t < t_0$, $g(t) = g_1(t)$ for $t > t_0$, $g(t_0) = hk + g_1(t_0)$

with $h > 0$, $1 > k > 0$

5.3.2.2

$$\text{So } P(x, t_0) = \begin{cases} e^{g_1(t_0)-g_1(x)} \text{with } \lim_{x\to t_0} P(x,t_0) = 1 \text{, in case (a)because} \\ g(t_0) = h + g_1(t_0),\ g(x) = h + g_1(x) \\ e^{g_1(t_0)-g_1(x)-h} \text{with } \lim_{x\to t_0} P(x,t_0) = e^{-h} \text{, in case (b)because} \\ g(t_0) = g_1(t_0),\ g(x) = h + g_1(x) \\ e^{g_1(t_0)-g_1(x)+h(k-1)} \text{with } \lim_{x\to t_0} P(x,t_0) = e^{h(k-1)} \text{, in case (c)because} \\ g(t_0) = hk + g_1(t_0),\ g(x) = h + g_1(x) \end{cases}$$

(see figg.12,13).

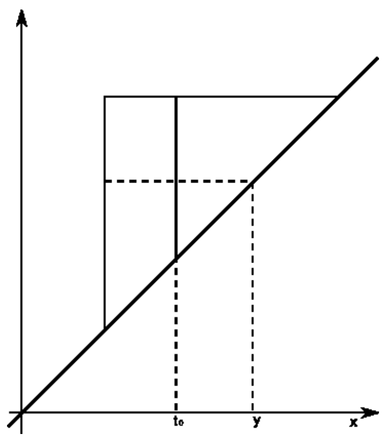

Fig. 12

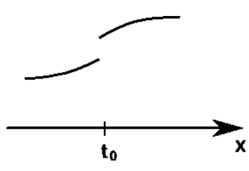

Fig. 13

5.3.2.3

$$\text{and } P(t_0 , y)= \begin{cases} e^{g_1(y)-g_1(t_0)-h} \text{with } \lim\limits_{y\to t_0} P(t_0 ,y) = e^{-h}, \text{in case (a)because} \\ \qquad g(t_0) = h + g_1(t_0),\ g(y) = g_1(y) \\ e^{g_1(y)-g_1(t_0)} \text{with } \lim\limits_{y\to t_0} P(t_0 ,y) = 1 \quad , \text{ in case (b)because} \\ \qquad g(t_0) = g_1(t_0),\ g(y) = g_1(y) \\ e^{g_1(y)-g_1(t_0)-hk} \text{with } \lim\limits_{y\to t_0} P(t_0 ,y) = e^{-hk}, \text{in case (c)because} \\ \qquad g(t_0) = hk + g_1(t_0),\ g(y) = g_1(y) \end{cases}$$

(see figg.14,15).

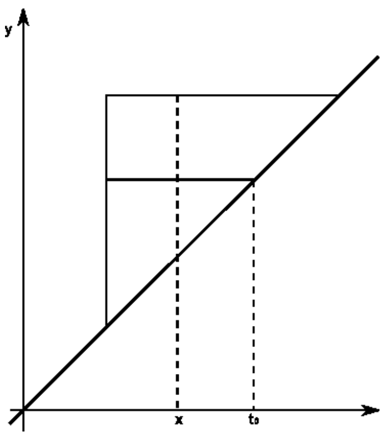

Fig. 14

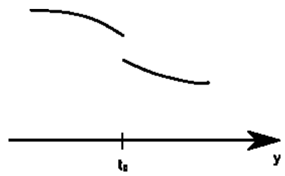

Fig. 15

5.3.2.4 We have $P(t_0 , t_0) = 1$ in cases (a) and (b) and $1 \geq P(t_0 , t_0) \geq \max(e^{-hk}, e^{h(k-1)})$ in case (c). We can then remark that the third type of discontinuity is reflected on the bisector while the first two are not: they are covered by the monotonicity of 2.1.3 and by discontinuity on one side only.

5.4.1 We shall now observe what happens to P(x,y) when $y= t_0$ $(x< t_0)$ and in its neighbourhood.

It is: $\lim\limits_{y\to t_0^-} g(y)= g_1(t_0) + h$, $\lim\limits_{y\to t_0^+} g(y)= g_1(t_0)$,

and according to the three cases (a) $g(t_0) = g_1(t_0)$, (b) $g(t_0) = g_1(t_0) + h$, (c) $g(t_0) = g_1(t_0) + hk$;

and then: $\lim\limits_{y\to t_0^-} P(x,y)= e^{g_1(t_0)+h-g_1(x)}$, $\lim\limits_{y\to t_0^+} P(x,y)= e^{g_1(t_0)-g_1(x)}$,

with $e^{g_1(t_0)+h-g_1(x)} > e^{g_1(t_0)-g_1(x)}$ as $e^h > 1$, and the amplitude the jump is $e^{g_1(t_0)-g_1(x)}(e^h - 1)$,

5.4.2 And now we shall observe what happens when $x= t_0$ $(y > t_0)$ and in its neighbourhood.

It is: $\lim\limits_{x\to t_0^-} g(x)= g_1(t_0) + h$, $\lim\limits_{x\to t_0^+} g(x)= g_1(t_0)$,

and according to the three cases (a) $g(t_0) = g_1(t_0)$, (b) $g(t_0) = g_1(t_0) + h$, (c) $g(t_0) = g_1(t_0) + hk$;

and then: $\lim\limits_{x\to t_0^-} P(x,y)= e^{g_1(y)-g_1(t_0)-h}$, $\lim\limits_{x\to t_0^+} P(x,y)= e^{g_1(y)-g_1(t_0)}$, with $e^{g_1(y)-g_1(t_0)} > e^{g_1(y)-g_1(t_0)-h}$, as $e^{-h} < 1$, and the amplitude the jump is $e^{g_1(t_0)-g_1(x)}(e^{h} - 1)$.

5.5 The final results on discontinuities show that the countable additivity principle is effectively restrictive.

6-**Some examples of use**

6.1 A first use is to end solving the problem outlined by Kendall(1948) and then addressed by Rossi(1976,779,79a) under at the time usual differentiability hypotheses, partially relaxed in (1979b). However we must remember that the theme treated here is more general

6.2 Another very important use can be the survival probability of a single individual already hinted to in 2.2.2.2 and also in 5.0.1, we can establish birth at 0, see that if we want to exclude the case of the born-dead we should not only impose P(0, 0) =1 but obtain an overall different P along x=0 at least, and not only see that P is null for negative x and for $y \geq \omega$, ω the extreme age, but also that P is discontinuous on the left for x=0 (and that discontinuity is not reflected elsewhere) if it is not everywhere null, while it can be continuous on $y = \omega$.

7-**Final consideration**

We addressed the problem of determining the probability that an isolated system remain unchanged over time with no restrictive hypotheses, based only on the nature of the problem and of probability. Former treatments of the problem were done under severe regularity conditions which made the conclusions not universally true as desirable though easy to attain. In particular countable additivity - and its smoothness consequences - was avoided as it is not necessary for the concept: it resulted that this assumption would be severely restrictive. The main tool has been solving a Sincov type functional equation with strong restrictions on the variables contrarily with the usual absence of any such restriction. The final universal result is that the enquired probability is the exponential of the increment along time of any decreasing everywhere defined one variable real function; and we feel that the problem is closed.

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