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PRICE REVEAL AUCTIONS

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Abstract

A price reveal auction is a Dutch auction in which the current price of the item on sale remains hidden. Bidders can privately observe the price only by paying a fee, and every time a bidder does so, the price falls by a predetermined amount. We solve for the perfect Bayesian equilibria of the game. If the number of participants n is common knowledge, then in equilibrium at most one bidder observes the price and the profits that the mechanism raises, if any, are only marginally higher than those that would stem from a normal sale. If instead n is a random variable then multiple entry can occur and profitability is enhanced.

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1 Introduction

This paper analyses a new and peculiar online selling mechanism, the so-called price reveal auctions. A price reveal auction is a Dutch auction (i.e., descending price) in which the current price of the item on sale is not publicly observable. Each bidder can privately observe the price by paying a fee c. The bidder is then given a limited amount of time (say, 10 seconds) to decide if he wants to buy the good at the current price. If the bidder buys the good, the auction ends. Otherwise, the price is decreased by a fixed amount $\Delta < c$ and the auction continues.[1] In other words, in a price reveal auction the price is hidden and falls by Δ every time a bidder observes it. Therefore, and contrary to standard Dutch auction procedures, the price does not fall exogenously at a predetermined speed, but rather endogenously in response to bidders' behavior.

Price reveal auctions are an example of the more general category of pay-per-bid auctions, a class of mechanisms in which bidders must pay the auctioneer a small fee every time they "move". These mechanisms enrich traditional auction formats with some original elements and have recently experienced a noticeable success on the Internet as well as attention from the media. The reason for this popularity is that items are usually sold for extremely low prices (often less than 5% of the market value). Nevertheless, these auction formats can turn out to be profitable for the seller, as the revenues that the latter collects through the bidding fees may more than compensate for the low selling price.

While the commercial success of pay-per-bid auctions is somehow fading away, the fact that these mechanisms attract the attention of academic researchers should not come as a surprise.[2] There are in fact many interesting aspects that charac-

[1] Obviously the decision to observe the price remains private so that no bidder can infer the current price by counting the number of times the price has been observed.

[2] Lowest unique bid auctions (LUBAs) and penny auctions are the most prominent examples

terize pay-per-bid auctions. First, these are games in which bidders face complex strategic situations whose equilibria are non trivial. Second, it is relatively easy to retrieve field and/or experimental data about bidders' actual behavior; this allows researchers to test theoretical predictions and identify other empirical regularities. Finally, the optimal design of these mechanisms is still unclear. The last point is important as these mechanisms may turn out to be useful in non commercial contexts. For instance, and thanks to their intrinsic fun component, pay-per-bid auctions can be used for charity purposes and/or fund-raising activities; i.e., situations in which people are more consciously willing to "lose some money".

In this paper we study the theoretical properties of price reveal auctions and define the perfect Bayesian equilibria of this mechanism under two different specifications. In the first setting, we let the number of participants n be common knowledge among all the players. In the second, we assume instead that bidders do not know n with certainty but do know its probability distribution.

The two scenarios present some similarities as well as some important differences. A common trait is that in both specifications an agent's optimal strategy is to observe the hidden price if and only if he believes that the latter is smaller than his private valuation net of the bidding fees. This result derives from the fact that the price decreases endogenously rather than exogenously and it implies that, despite the apparent similarities with a standard Dutch auction, the bidding behavior of a price reveal auction is more similar to the one that characterizes second price auctions rather than first price auctions (see for instance Krishna, 2002). Differences

of pay-per-bid auctions. In a LUBA, bidders place private bids and the winner is the agent who submits the lowest offer that is not matched by any other bid. Theoretical and/or empirical analysis of LUBAs appears in Eichberger and Vinogradov (2008), Gallice (2009), Rapoport *et al.* (2009), Houba *et al.* (2011), and Östling *et al.* (2011). In a penny auction each bid increases the current price by a fixed amount (a penny) and restarts a public countdown; the winner is the bidder who holds the winning bid when the countdown expires. Various aspects of penny auctions are analyzed in Augenblick (2009), Byers *et al.* (2010), Hinnosaar (2010) and Platt *et al.* (2010).

between the two specifications subsist instead for what concerns possible equilibrium outcomes as well as the profitability of the mechanism. The (un)certainty about n in fact influences players' beliefs about the initial price that the seller sets and thus their decision to observe it or not.

We show that if n is common knowledge then in equilibrium the hidden price is observed at most only once by a single bidder. A price reveal auction thus either immediately finishes (i.e., an agent observes the price and buys the item as soon as the auction opens) or reaches the final period with no player ever observing the price (i.e., the item remains unsold). When the mechanism is able to generate profits for the seller (which is not always the case), these are only marginally higher than the profits that would be realized in a normal market transaction.

Conversely, we show that multiple entries can occur along the equilibrium path when n is a random variable. The uncertainty about n implies uncertainty about the hidden price. As such, a bidder may rationally decide to observe the price, discover an actual price that is higher than expected, and thus refuse to buy the item. If this is the case, the selling price indeed decreases, but thanks to the accrual of bidding fees the overall profitability of the mechanism is enhanced.

2 The model

We model the sale of a single indivisible item through a price reveal auction. There are $n+1$ risk-neutral players: a seller (indexed by s) and a finite set $N = \{1, ..., n\}$ of potential buyers. We will investigate two different scenarios: in the first scenario (Section 3), the number of participants n is common knowledge among all players. In the second scenario (Section 4) buyers only know that n is a random variable that follows the non-degenerate probability distribution g.

The seller's valuation for the good on sale is given by $v_s = v_r$ where v_r is the publicly known retail price.[3] Each buyer has a valuation v_i that is independently and identically distributed on the interval $[0, \bar{v}]$ according to the cumulative distribution function F, which is strictly increasing and continuously differentiable with density f and such that $\bar{v} \geq v_r$. In line with the standard independent private value assumption, bidders know their type v_i and all players know that every $v_{j \neq i}$ is drawn from F.

Time is discrete and goes from $t = 0$ to $t = T$, where T is finite and common knowledge. At $t = 0$ the seller sets the initial price $p_0 \in [0, \bar{v}]$. The price p_0, and the current price p_t for any $t \in \{0, ..., T\}$, is unknown to potential buyers unless agents explicitly decide to observe it. At any period $t \in \{1, ..., T\}$ each agent $i \in N$ plays $a_{i,t} \in \{\emptyset, \gamma(\cdot)\}$. Action $a_{i,t} = \emptyset$ indicates that player i remains inactive. Action $\gamma(\cdot) : [0, \bar{v}] \rightarrow \{0, 1\}$ indicates that agent i observes $p_t \in [0, \bar{v}]$ and then decides to buy the good ($\gamma(p_t) = 1$) or not ($\gamma(p_t) = 0$). The agent is charged the fee $c > 0$ whenever he observes p_t. If multiple players simultaneously play $a_{i,t} = \gamma(\cdot)$, we assume that the seller randomly selects a single agent to whom he discloses p_t and charges c.

Every time a buyer plays $a_{i,t} = \gamma(\cdot)$, the price decreases from p_{t-1} to $p_t = p_{t-1} - \Delta$ with $\Delta \in (0, c)$.[4] Otherwise, $p_t = p_{t-1}$. The auction ends at $t_e \in \{1, ..., T\}$ where $t_e = T$ if $a_{i,t} = \emptyset$ or $a_{i,t} = \gamma(\cdot)$ with $\gamma(p_t) = 0$ for any i and any t while $t_e = \hat{t}$ as soon as a buyer plays $a_{i,\hat{t}} = \gamma(\cdot)$ with $\gamma(p_{\hat{t}}) = 1$ at period $\hat{t} \in \{1, ..., T\}$.

Finally, let $\eta_{i,t} \in \mathbb{N}_0$ be the number of times bidder i observes the price (and thus pays the fee c) up to period t included. Players' payoffs thus take the following

[3] We assume that $v_s = v_r$ in order to see if the mechanism is able to generate profits in excess with respect to the gains from trade that would be realized by a seller that operates in a standard market where consumers face the retail price v_r.

[4] In actual price reveal auctions the relation $\Delta = \frac{1}{2}c$ usually holds. Notice that if $\Delta > c$ then a bidder could drive p_t down to zero by playing $\frac{p_0}{\Delta}$ times action $a_{i,t} = \gamma(\cdot)$ with $\gamma(p_t) = 0$ for any $p_t > 0$. This strategy would cost $\frac{c}{\Delta} p_0$ and would thus ensure positive profits as far as $\frac{c}{\Delta} p_0 < v_i$.

form:[5]

$$u_{i,t_e} = \begin{cases} v_i - p_{t_e} - c\eta_{i,t_e} & \text{if } i \text{ buys the good} \\ -c\eta_{i,t_e} & \text{otherwise} \end{cases} \qquad \text{for } i \in N$$

$$u_{s,t_e} = \begin{cases} p_{t_e} - v_s + c\sum_{i\in N} \eta_{i,t_e} & \text{if there exists an } i \text{ that buys the good} \\ c\sum_{i\in N} \eta_{i,t_e} & \text{otherwise} \end{cases}$$

A price reveal auction is thus an extensive-form game with imperfect information, as bidders do not always know p_t and do not observe rivals' types and actions. As a solution concept, we apply the notion of (symmetric) perfect Bayesian equilibrium. Before properly defining such equilibrium (Proposition 1 for the case in which n is common knowledge, Proposition 2 for the case in which n is a random variable), we first discuss some of its characteristics and introduce some additional notations.

The rules of the game are such that buyers accumulate sunk costs c every time they observe the price. Therefore, an agent would ideally observe p_t only once, discover a price that he likes, and buy the item. In other words, the buyer should observe the price as soon as he is confident that, starting from p_{t-1}, one more observation would make p_t reach (or get smaller than) his willingness to pay $\beta(v_i)$. The latter is given by $\beta(v_i) = b(v_i) - c + \Delta$ where $b(v_i)$ is the agent's bidding function that will be identified in equilibrium. Notice that the agent internalizes the fact that if he observes the price at period t then he is charged c and the price moves from p_{t-1} to $p_t = p_{t-1} - \Delta$. As such he adjusts his willingness to pay accordingly and $\beta(v_i) < b(v_i)$.[6]

Let $\mu_{i,t}(\cdot)$ indicate agent i's beliefs at time t. More precisely, $\mu_{i,t}(\cdot)$ is the agent's

[5] We assume that the auctioneer buys the auctioned item (and thus pay $v_s = v_r$) on demand; i.e., only if the auction happens to have a winner.

[6] Say that $c = 2$, $\Delta = 1$, and agent i with valuation v_i is willing to invest up to $b(v_i) = 100$. The agent should then observe the price when he believes that $p_{t-1} \leq \beta(v_i)$ where $\beta(v_i) = 99$. In fact, if his beliefs are correct, the agent discovers the price $p_t \leq 98$, buys the item, and bears total costs of $p_t + 2 \leq 100 = b(v_i)$.

assessment about the probability of the realization of the event $(\cdot)$. A player must thus abstain from observing the price (i.e., play $a_{i,t} = \emptyset$) whenever $\mu_{i,t}(p_{t-1} \leq \beta(v_i)) \leq \tilde{\mu}_{i,t}$ where $\tilde{\mu}_{i,t}$ is a threshold that we will shortly define. In contrast, the agent observes the price if $\mu_{i,t}(p_{t-1} \leq \beta(v_i)) > \tilde{\mu}_{i,t}$. If this is the case, then the decision to buy the item or not depends on how the actual price p_t that the agent discovers compares with his willingness to pay (adjusted to take into account that in moving from p_{t-1} to p_t the price decreased by Δ and thus the agent compares p_t with $\beta(v_i) - \Delta = b(v_i) - c$). Potential buyers should then play according to the following rule:

$$a_{i,t} = \begin{cases} \emptyset & \text{if } \mu_{i,t}(p_{t-1} \leq \beta(v_i)) \leq \tilde{\mu}_{i,t} \\ \gamma(p_t) = 0 & \text{if } \mu_{i,t}(p_{t-1} \leq \beta(v_i)) > \tilde{\mu}_{i,t} \text{ and } p_t > b(v_i) - c \\ \gamma(p_t) = 1 & \text{if } \mu_{i,t}(p_{t-1} \leq \beta(v_i)) > \tilde{\mu}_{i,t} \text{ and } p_t \leq b(v_i) - c \end{cases} \tag{1}$$

To solve for the threshold of bidders' beliefs $\tilde{\mu}_{i,t}$, let $E_{i,t}(\cdot)$ denote the expectation operator and notice that if agent i decides to observe the price then $E_{i,t}(p_t) = E_{i,t}(p_{t-1}) - \Delta$. Consider then the condition:

$$\tilde{\mu}_{i,t}(v_i - E_{i,t}(p_t) - c) - (1 - \tilde{\mu}_{i,t})c = 0 \tag{2}$$

that, under the assumption of risk neutrality, imposes indifference between the expected payoff associated with the decision to observe the price (action $a_{i,t} = \gamma(\cdot) : [0, \bar{v}] \to \{0, 1\}$) and the decision to remain inactive (action $a_{i,t} = \emptyset$). Solving for $\tilde{\mu}_{i,t}$ one gets $\tilde{\mu}_{i,t} = \frac{c}{v_i - E_{i,t}(p_t)}$. Still notice that $\tilde{\mu}_{i,t} \in [0, 1]$ if and only if $v_i - E_{i,t}(p_t) \geq c$, i.e., $v_i - c \geq E_{i,t}(p_t)$. If this is not the case (and thus agent i expects to realize a negative payoff even if he buys the good) then $\tilde{\mu}_{i,t} = 1$ and the condition that triggers entry never verifies. In sum:

$$\tilde{\mu}_{i,t} = \begin{cases} \frac{c}{v_i - E_{i,t}(p_t)} & \text{if } v_i - c \geq E_{i,t}(p_t) \\ 1 & \text{otherwise} \end{cases} \tag{3}$$

3 The equilibrium when n is known

Assume that the number of potential bidders n is common knowledge among all the players. Then in equilibrium a price reveal auction attracts at most one active bidder and the hidden price is observed at most once. Proposition 1 formalizes this result, but the intuition is that the price could be observed multiple times if and only if bidders have non-deterministic beliefs about it. In fact, the only possibility for the price to be observed more than once is that some bidders observe p_t, discover a price that is higher than expected, and thus refuse to buy the item. But Proposition 1 shows that the fact that n is common knowledge removes the uncertainty about the initial price p_0^* that the seller sets[7]. Bidders' beliefs about p_0^*, as well as about how the price evolves over time, are thus deterministic. This implies that action $a_{i,t} = \gamma(\cdot) : [0, \bar{v}] \rightarrow \{0, 1\}$ with $\gamma(p_t) = 0$ is strictly dominated.[8] As such, the choice of playing it can only stem from inconsistent beliefs and/or a mistake and thus cannot be part of a Bayesian equilibrium.

Proposition 1 *In the unique symmetric Bayesian perfect equilibrium of a price reveal auction where the number of participants n is common knowledge, bidders know that the seller sets the initial price:*

$$p_0^* = \underset{p_0 \in [v_r + \Delta - c, \bar{v} + \Delta - c]}{\arg\max} \left(1 - \left[F\left(p_0 - \Delta + c\right)\right]^n\right)\left(p_0 - \Delta + c - v_r\right)$$

[7]Here and in what follows we use the asterisk * to indicate equilibrium prices, actions, and beliefs.

[8]More precisely, and given that $\Delta < c$, $a_{i,t} = \gamma(\cdot) : [0, \bar{v}] \rightarrow \{0, 1\}$ with $\gamma(p_t) = 0$ is dominated by $\gamma(\cdot) : [0, \bar{v}] \rightarrow \{0, 1\}$ with $\gamma(p_t) = 1$ whenever $\mu_{i,t}(p_{t-1}^* \leq \beta^*(v_i)) = 1$ and by $a_{i,t} = \emptyset$ whenever $\mu_{i,t}(p_{t-1}^* \leq \beta^*(v_i)) = 0$.

and thus in any $t \in \{1, ..., t_e\}$ *they play* $\left(a^*_{i,t}\right)_{t=1}^{t_e}$*, where* $a^*_{i,t}$ *is such that:*

$$a^*_{i,t} = \begin{cases} \emptyset & \text{if } \mu^*_{i,t}(p_{t-1} \leq \beta^*(v_i)) \leq \tilde{\mu}_{i,t} \\ \gamma(p_t) = 0 & \text{if } \mu^*_{i,t}(p_{t-1} \leq \beta^*(v_i)) > \tilde{\mu}_{i,t} \text{ and } p_t > b^*(v_i) - c \\ \gamma(p_t) = 1 & \text{if } \mu^*_{i,t}(p_{t-1} \leq \beta^*(v_i)) > \tilde{\mu}_{i,t} \text{ and } p_t \leq b^*(v_i) - c \end{cases}$$

with $\gamma(\cdot) : [0, \bar{v}] \rightarrow \{0, 1\}$*,* $\beta^*(v_i) = b^*(v_i) - c + \Delta$*,* $b^*(v_i) = v_i$*,* $\tilde{\mu}_{i,t}$ *is as in (3) with* $E_{i,t}(p_{t-1}) = p^*_0$ *at any* $t \geq 1$*, and* $t_e \in \{1, T\}$*. At any* $t \in \{1, ..., t_e\}$ *bidders' beliefs are given by* $\mu^*_{i,t}(p_{t-1} \leq \beta^*(v_i)) = 1$ *if* $p^*_0 \leq \beta^*(v_i)$ *and* $\mu^*_{i,t}(p_{t-1} \leq \beta^*(v_i)) = 0$ *if* $p^*_0 > \beta^*(v_i)$*.*

Proof. In the appendix. ■

Notice that as far as p^*_0 is unique (i.e., the expected profit function of the seller has a unique maximizer) then the equilibrium defined by Proposition 1 is unique and in pure strategies. The proposition also implies that on the equilibrium path only two situations can arise: either there exists an agent who observes the price in $t = 1$, and immediately buys the item; or no agent ever observes the price and the item remains unsold. In the first case the mechanism raises positive profits; in the second it raises zero profits. Which of these two equilibrium outcomes occurs depends on how the actual realization of agents' types combines with the initial price p^*_0 set by the auctioneer. The following example illustrates the two possibilities:

Example 1 *Consider a price reveal auction with* $v_s = v_r = 100$*,* $c = 2$*,* $\Delta = 1$*,* $N = \{1, 2, 3\}$ *and let* F *be uniform on* $[0, 150]$*. The seller sets* p^*_0 *in order to maximize his expected payoff* $E_{s,0}\left(u_{s,t_e}\right) = \left(1 - \left[\frac{1}{150}\left(p_0 + 2\right)\right]^3\right)\left(p_0 - 99\right)$*, i.e.,* $p^*_0 \simeq 125.58$ *and* $E_{s,0}\left(u_{s,t_e} \mid p^*_0\right) \simeq 10.23$*.*

Now say that types' actual realizations are $v_1 = 40$*,* $v_2 = 100$*, and* $v_3 = 140$*. Then* $\beta^*(v_1) = 39$*,* $\beta^*(v_2) = 99$*, and* $\beta^*(v_3) = 139$*. It follows that* $a^*_{i,1} = \emptyset$ *for* $i \in \{1, 2\}$

and $a^*_{3,1} = \gamma(\cdot) : [0, 150] \to \{0, 1\}$ *with* $\gamma(p^*_0) = 1$*. The auction ends at* $t_e = 1$ *and payoffs are* $u_{i,1} = 0$ *for* $i \in \{1, 2\}$*,* $u_{3,1} \simeq 13.42$*, and* $u_{s,1} \simeq 26.58$*.*
If instead agent 3*'s type is* $v'_3 = 120$ *then* $\beta^*(v'_3) = 119$ *and* $\left(a^*_{i,t}\right)^T_{t=1} = \emptyset$ *for any* $i \in N$*. The auction ends at* $t_e = T$*, the item remains unsold, and payoffs are* $u_{s,T} = 0$ *and* $u_{i,T} = 0$ *for any* $i \in N$*.*

When the mechanism generates profits, these are only marginally higher than the profits that would stem from a normal market transaction. If a seller with valuation v_r uses a price reveal auction to sell to an agent with valuation v_i an item at the price p^*_0, then he makes profits $u_{s,1} = p^*_0 - \Delta - v_r + c$. On the contrary, if the seller sells the item through a normal sale then profits amount to $u'_{s,1} = p^*_0 - v_r$. It follows that $u_{s,1} = u'_{s,1} + (c - \Delta)$ and therefore, given that $c > \Delta$, $u_{s,1} > u'_{s,1}$. Still, this (very moderate) extra profitability of a price reveal auction does not come for free: in fact, an agent with valuation $v_i \in (p^*_0, p^*_0 + (c - \Delta))$ does not enter a price reveal auction but would instead buy the item via a normal sale.[9]

Going back to Proposition 1, an interesting feature of the equilibrium is that $b^*(v_i) = v_i$; i.e., bidders bid their types. Agents' behavior thus resembles the one that characterizes second price auctions and a bidder's willingness to pay ($\beta^*(v_i) = v_i - c + \Delta$) can be interpreted as his personal valuation net of the net cost of observing the hidden price. In particular, in equilibrium bidders do not shade their bids as they would do in first price auctions (and thus in a standard Dutch auction). The reason for this is that on the equilibrium path the price does not fall as it actually remains constant at the level $p^*_t = p^*_0$. Agents thus realize that the initial price cannot reach a better level and decide what to do by comparing the payoff they

[9] To put things in a different way, a price reveal auction in which the seller sets the initial price p^*_0 appeals to the same consumers and raises the same profits as a normal sale in which the price of the item is given by $p^*_0 + (c - \Delta)$.

would get if they buy at $t = 1$ ($u_{i,1} = v_i - p_0^* + \Delta - c$) with the payoff they would get if they do not participate ($u_{i,T} = 0$).

4 The equilibrium when n is unknown

Contrary to the previous section, assume now that the number of potential bidders n is not common knowledge among all the players. More precisely, the seller still knows n with certainty while bidders do not. Each bidder only knows that $n = \hat{n} + 1$ where $\hat{n}$ (i.e., the number of other participants) is a discrete random variable distributed over the support $\{0, ..., \bar{n}\}$ according to the non-degenerate probability distribution g. This different information structure is certainly more appropriate to capture the features of price reveal auctions that take place over the Internet, where players do not know the number of bidders. It also has important implications for what concerns possible equilibrium outcomes and the profitability of the mechanism.

The uncertainty about n implies uncertainty about the initial price p_0^* set by the seller. And the uncertainty about p_0^* in turn implies uncertainty about the current price p_t^* at any $t \geq 1$. In such an environment, agents still play according to the behavioral rule defined in (1) but now action $a_{i,t} = \gamma(\cdot) : [0, \bar{v}] \rightarrow \{0, 1\}$ with $\gamma(p_t) = 0$ can be observed even on the equilibrium path. At any period, agents decide whether or not to observe the price by comparing their expectations about the hidden price with their willingness to pay. As such, it may well be the case that an agent rationally decides to observe p_t, discovers an actual price that is higher than expected, and thus decides not to buy the item. This implies that the hidden price may actually fall. Nevertheless, in equilibrium agents still bid their true valuation ($b^*(v_i) = v_i$) as this possible price decrease is endogenous and uncertain as it depends on the moves (actually the mistakes) of the other players. The following proposition formalizes these results:

Proposition 2 *In the unique symmetric Bayesian perfect equilibrium of a price reveal auction, in which bidders do not know the number of participants with certainty, bidders expect the auctioneer to set the initial price at the level:*

$$E_{i,0}(p_0^*) = \sum_{\hat{n}=0}^{\hat{n}=\bar{n}} g(\hat{n}) \left(\underset{p_0 \in [v_r+\Delta-c, \bar{v}+\Delta-c]}{\arg\max} \left(1 - \left[F\left(p_0 - \Delta + c\right)\right]^{\hat{n}+1}\right)\left(p_0 - \Delta + c - v_r\right) \right)$$

and therefore at any $t \in \{1, ..., t_e\}$ *they play* $\left(a_{i,t}^*\right)_{t=1}^T$, *where* $a_{i,t}^*$ *is such that:*

$$a_{i,t}^* = \begin{cases} \emptyset & \text{if } \mu_{i,t}^*(p_{t-1} \leq \beta^*(v_i)) \leq \tilde{\mu}_{i,t} \\ \gamma(p_t) = 0 & \text{if } \mu_{i,t}^*(p_{t-1} \leq \beta^*(v_i)) > \tilde{\mu}_{i,t} \text{ and } p_t > b^*(v_i) - c \\ \gamma(p_t) = 1 & \text{if } \mu_{i,t}^*(p_{t-1} \leq \beta^*(v_i)) > \tilde{\mu}_{i,t} \text{ and } p_t \leq b^*(v_i) - c \end{cases}$$

with $\gamma(\cdot) : [0, \bar{v}] \rightarrow \{0, 1\}$, $\beta^*(v_i) = b^*(v_i) - c + \Delta$, $b^*(v_i) = v_i$, $\tilde{\mu}_{i,t}$ *is as in (3), and* $t_e \in \{1, ..., T\}$. *Bidders' initial beliefs are given by* $\mu_{i,1}^*(p_0 \leq \beta^*(v_i)) \in \left[\frac{1}{2}, 1\right)$ *if* $E_{i,0}(p_0^*) \leq \beta^*(v_i)$ *and* $\mu_{i,1}^*(p_0 \leq \beta^*(v_i)) \in \left(0, \frac{1}{2}\right)$ *if* $E_{i,0}(p_0^*) > \beta^*(v_i)$.

Proof. In the appendix. ■

A price reveal auction is thus now characterized by three possible (classes of) equilibrium outcomes: 1) a unique bidder observes the price, buys the item, and the auction closes immediately; 2) no bidder ever observes the price, the auction closes at $t = T$, and the item remains unsold; 3) more than one bidder observes the price and the auction ends at $t_e \in \{2, ..., T\}$ if a player buys the item at t_e or at $t_e = T$ if no bidder buys it.

The first two outcomes are analogous to the ones that characterize a price reveal auction in which n is common knowledge; the third one is instead specific of a price reveal auction where there is uncertainty about the number of participants. In this third equilibrium outcome, entry of multiple bidders and repeated observations of the price can occur. As already mentioned, this is due to the fact that players do not hold deterministic beliefs about the hidden price. Notice that Proposition 2 does not exactly pin down agents' initial beliefs (it only puts some bounds). Neither does it trace how these beliefs evolve over time. Still, this does not undermine the validity of the results: uncertainty about the current price and the non-deterministic nature of agents' beliefs are necessary conditions to potentially trigger multiple entry.

Multiple entry positively affects the profitability of the mechanism as seller's profits increase by $(c - \Delta) > 0$ every time an agent observes the price. Therefore, a price reveal auction with uncertainty about the number of participants can be more profitable than an auction in which n is common knowledge. Notice that agents observe the price only when they are confident that they will buy the item (i.e., conditional on having decided to observe the price, the event of buying the item is more likely than the event of not buying it). As such, long sequences of agents who observe the price and refuse to buy the item, while theoretically possible, become progressively less and less likely and the expected profitability of a price reveal auction thus appears pretty limited.[10]

5 Robustness of the equilibria

The equilibria identified in Propositions 1 and 2 are robust to a number of generalizations:

Random arrival of bidders. Assume a new bidder j becomes aware of the auction at a certain period $t' > 1$. If $t' > t_e$, then the auction closes and nothing changes in the equilibrium strategies played by the incumbent players. But even if $t' \leq t_e$ Propositions 1 and 2 continue to hold. Consider first the case in which the number

[10] Moreover, in discussing the profitability of price reveal auctions, we have not so far mentioned the fixed and variable costs that a seller incurs in order to run the business (for instance, the launch and maintenance of a web platform that implements the mechanism).

of initial players n is common knowledge: player j (the $(n+1)th$ bidder) observes that the auction is still open at t' and thus correctly sets $p_{t'} = p_0^*$ as defined in Proposition 1 and then play accordingly: he immediately observes the price and buys the item if $p_0^* \leq \beta^*(v_j)$, while he remains idle and plays $a_{j,t} = \emptyset$ for any $t \geq t'$ otherwise. As such, the arrival of new bidders can change t_e, but neither modifies the structure of the equilibrium, nor makes the mechanism more profitable. Similarly, if n is a random variable then the player j computes $E_{j,0}(p_0^*)$ as defined in Proposition 2 and then plays according to equilibrium. Notice that, in this case, the arrival of a new bidder can modify the profitability of the mechanism. More precisely, a seller's profits can be enhanced (the new entrant observes the price and decides not to buy the item), penalized (the entrant observes the price and buys the item pre-empting future observations of other players), or unaffected (the entrant does not observe the price).

Risk attitudes of the bidders. Equilibrium behavior and the profitability of the mechanism do not change if (some or all of) the potential buyers are risk-lovers rather than risk-neutral.[11] In fact, it is a well-known result (see, for instance, Krishna, 2002) that in both cases $b^*(v_i) = v_i$ as risk attitudes do not modify the optimal bidding strategy in a second price auction which, as we saw, is the category to which price reveal auctions belong.

Strategic choice of c. If the seller could simultaneously choose the initial price $p_0^* \in [0, \bar{v}]$, as well as the fee $c^* \in (0, p_0^*)$, the qualitative features of the equilibria would not change. In particular, buyers would hold the same beliefs and implement the same strategies. As for the choice of c^*, the auctioneer would have to find the balance between two conflicting forces: on one hand, a higher c^* (holding fixed Δ)

[11] The hypothesis of risk loving behavior for some or all bidders seems appropriate given the peculiar features of a price reveal auction and the self-selection of the participants.

increases profits in case entry occurs; on the other hand, a higher c^* decreases $\beta^*(v_i)$ and thus makes more likely the equilibrium outcome in which no agent observes the price and the item remains unsold.

6 Conclusions

Pay-per-bid auctions are a new form of selling mechanisms that have been experiencing a noticeable success on the Internet. But apart from their commercial use, and thanks to their intrinsic "fun" component, pay-per-bid auctions may also find applications in other contexts such as charities and fund-raising activities. As such, the design of these mechanisms raises a number of interesting theoretical questions.

In this paper we have analyzed the most recent example of a pay-per-bid mechanism, the so-called price reveal auction. We showed that if agents are fully rational and the number of participants is common knowledge, then a price reveal auction attracts, at most, one active bidder and, at best, raises profits that are only marginally higher than those that would stem from a normal market transaction. In a more common case in which there is uncertainty about the number of players, a price reveal auction can instead trigger multiple entry even on the equilibrium path. Because of the accrual of players' bidding fees, multiple entry enhances the expected profitability of the mechanism; the latter in any case remains limited.

The commercial experience of price reveal auctions seems to be consistent with these theoretical results. Indeed, the life cycle of a number of websites that implemented this auction mechanism has been quite extemporaneous. In the very short run these websites, possibly also exploiting consumers' enthusiasm and naiveté, managed to be profitable and flourish. But this profitability rapidly decreased such that in the space of very few months the vast majority of these websites closed down.

Appendix

Proof of Proposition 1

Conditional on the subsequent behavior of the bidders, p_0^* maximizes the seller's expected profits $E_{s,0}(u_{s,t_e})$. These are given by:

$$E_{s,0}(u_{s,t_e}) = \left(1 - \left[F\left(b^{*^{-1}}(p_0 - \Delta + c)\right)\right]^n\right)(p_0 - \Delta + c - v_r) \quad (4)$$

The first term on the RHS of (4) is the probability that there exists at least one bidder whose valuation v_i is such that $v_i \geq v_{\tilde{\imath}}$ where $v_{\tilde{\imath}}$ is the valuation of an hypothetical agent $\tilde{\imath}$, whose willingness to pay is p_0 (i.e., an agent who would be indifferent between buying the good at $t = 1$ at price $p_1 = p_0 - \Delta$ and not entering the auction). More formally, $\beta^*(v_{\tilde{\imath}}) = p_0$ implies $b^*(v_{\tilde{\imath}}) - c + \Delta = p_0$ and thus $v_{\tilde{\imath}} = b^{*^{-1}}(p_0 - \Delta + c)$. The second term on the RHS of (4) is the payoff the seller realizes if he sells the good at price $p_1 = p_0 - \Delta$; i.e., to the first agent who observes the price. Given $E_{s,0}(u_{s,t_e})$, p_0^* is thus given by:

$$p_0^* = \underset{p_0 \in [v_r + \Delta - c, b^*(\bar{v}) + \Delta - c]}{\arg\max} \left(1 - \left[F\left(b^{*^{-1}}(p_0 - \Delta + c)\right)\right]^n\right)(p_0 - \Delta + c - v_r) \quad (5)$$

The lower bound of the interval $[v_r + \Delta - c, b^*(\bar{v}) + \Delta - c]$ ensures that the payoff is non-negative while the upper bound is required for the probability to be well-defined. The function $E_{s,0}(u_{s,t_e})$ is continuous and the interval, which we will show that in equilibrium always exists, is closed and bounded. It follows that a maximizer p_0^* certainly exists, and if p_0^* is unique then the equilibrium is unique. In fact, $E_{s,0}(u_{s,t_e} \mid p_0^*) > E_{s,0}(u_{s,t_e} \mid p_0')$ for any $p_0' \neq p_0^*$ such that it cannot exist an equilibrium where the seller sets a different initial price.

Given p_0^*, buyers' behavior is optimal given their beliefs and these beliefs are consistent. In particular, a bidder j for whom $\beta^*(v_j) \geq p_0^*$ plays $a_{j,1}^* = \gamma(\cdot) : [0, \bar{v}] \rightarrow \{0, 1\}$ and $\gamma(p_1^*) = 1$ where $p_1^* = p_0^* - \Delta$.[12] On the contrary, an agent for which $\beta^*(v_i) < p_0^*$ plays $a_{i,1}^* = \emptyset$. The latter remains an equilibrium for every t given that $\Delta < c$; i.e., the private benefits of observing p_t are smaller than the cost. If the auction is still open at generic period $\tilde{t} > 1$, bidders correctly infer that $a_{i,t}^* = \emptyset$ for any i and any $t \in \left\{1, ..., \tilde{t} - 1\right\}$ and thus $p_{\tilde{t}-1} = p_0^*$. It follows that $\mu_{i,\tilde{t}}(p_{\tilde{t}-1} \leq \beta^*(v_i)) = 0$ and $a_{i,\tilde{t}}^* = \emptyset$ for any i.

Finally, given that $u_{i,T} = 0$ is the payoff of a bidder who plays $(a_{i,t}^* = \emptyset)_{t=1}^T$ and that on the equilibrium path $p_{t-1}^* = p_0^*$ for any t, it follows that $b^*(v_i) = v_i$. Every $b'(v_i) \neq v_i$ is in fact (weakly) dominated. First, let $b'(v_i) > v_i$: strategies $b'(v_i)$ and $b^*(v_i)$ lead to the same payoff unless $b'(v_i) > p_0^* + c - \Delta > b^*(v_i)$. If this is the case, a bidder who plays $b^*(v_i)$ does not enter the auction and gets $u_{i,T} = 0$, while a bidder who plays $b'(v_i)$ observes p_0^* and then either gets $u'_{i,1} = v_i - p_0^* + \Delta - c < 0$ (the agent buys the item) or $u'_{i,1} = -c$ (the agent does not buy the item). Alternatively, let $b'(v_i) < v_i$: strategies $b'(v_i)$ and $b^*(v_i)$ lead to the same payoff unless $b^*(v_i) > p_0^* + c - \Delta > b'(v_i)$. If this is the case, a bidder who plays $b^*(v_i)$ observes p_0^*, buys the item, and gets $u_{i,1} = v_i - p_0^* + \Delta - c > 0$ while a bidder who plays $b'(v_i)$ does not enter the auction and gets $u_{i,T} = 0$.

By substituting $b^*(v_i) = v_i$ in (5) one gets:

$$p_0^* = \underset{p_0 \in [v_r + \Delta - c, \bar{v} + \Delta - c]}{\arg\max} \left(1 - \left[F\left(p_0 - \Delta + c\right)\right]^n\right)\left(p_0 - \Delta + c - v_r\right)$$

and the interval for p_0 always exists given that $\bar{v} \geq v_r$. □

[12] As already mentioned in the text, if multiple agents of type j simultaneously want to observe the price then the seller randomly selects one of these who pays c, observes $p_1^* = p_0^* - \Delta$, and buys the item.

Proof of Proposition 2

The proof is essentially analogous to the proof of Proposition 1. Therefore, we only highlight the differences.

Given that n is not common knowledge, bidders cannot retrieve the initial price p_0^* in a deterministic way. They instead rely on the expected initial price $E_{i,0}\left(p_0^*\right)$, which is given by the weighted average of the profit-maximizing initial prices that would be chosen by the seller for any possible realization of the random variable $\hat{n}$. As such:

$$E_{i,0}(p_0^*) = \sum_{\hat{n}=0}^{\hat{n}=\bar{n}} g(\hat{n}) \left(\underset{p_0 \in [v_r+\Delta-c, b^*(\bar{v})+\Delta-c]}{\arg\max} \left(1 - \left[F\left(b^{*^{-1}}\left(p_0 - \Delta + c\right)\right)\right]^{\hat{n}+1}\right)\left(p_0 - \Delta + c - v_r\right)\right)$$

The bounds on players' initial beliefs follow from the fact that if $E_{i,0}(p_0^*) \leq \beta^*(v_i)$, i.e., the agent's willingness to pay is larger than the initial price the agent expects the seller to set, then the probability that the agent associates to the event $p_0^* \leq \beta^*(v_i)$ must be at least $\frac{1}{2}$. In other words the agent places $\beta^*(v_i)$ above the mean $E_{i,0}(p_0^*)$, i.e., in the right part of the probability distribution of p_0^*. Similar considerations lead to state that if $E_{i,0}(p_0^*) > \beta^*(v_i)$ then $\mu_{i,1}^*(p_0^* \leq \beta^*(v_i)) \in \left(0, \frac{1}{2}\right)$.

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