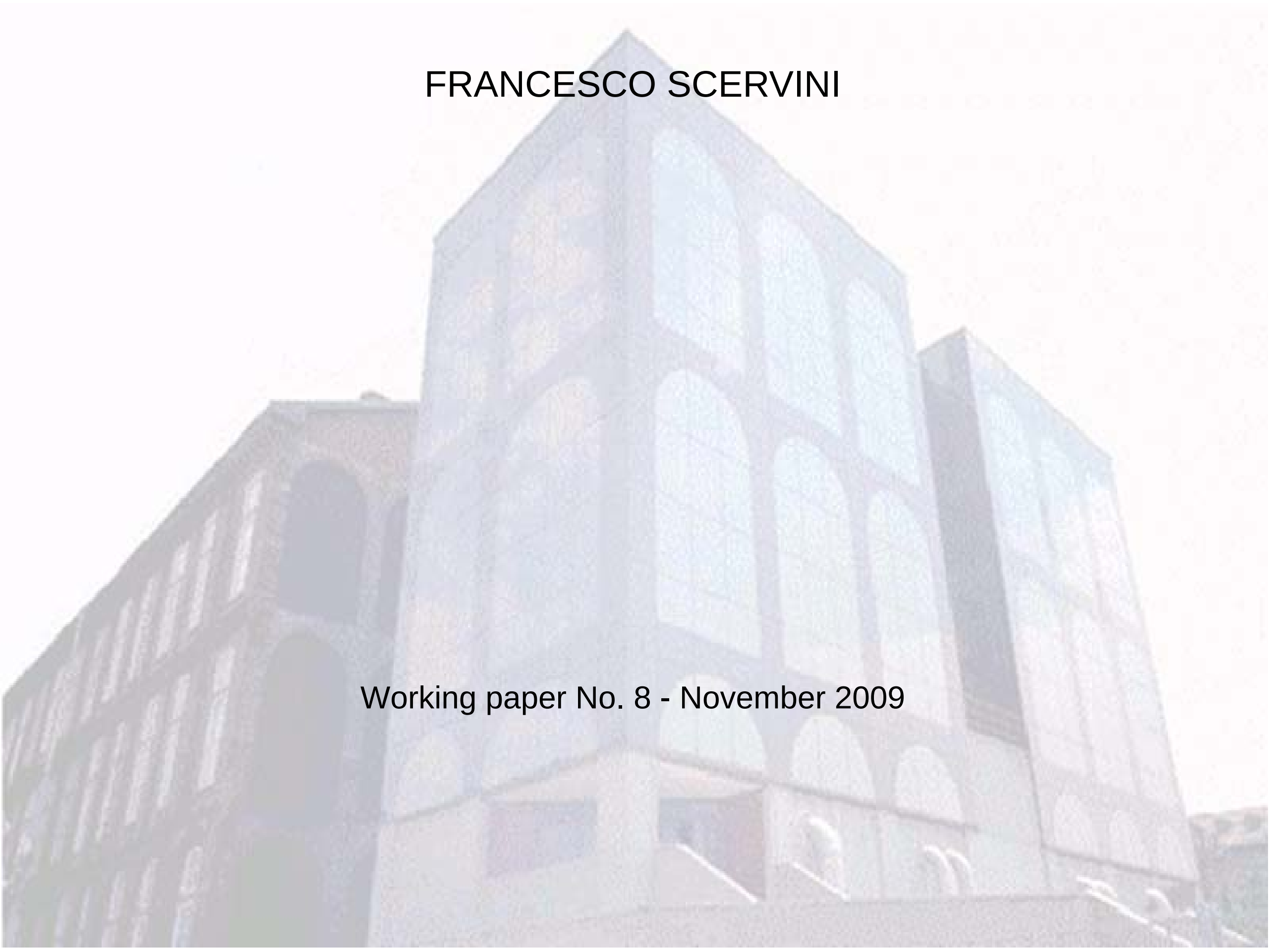
UNIVERSITÀ
DEGLI STUDI
DI TORINO

ALMA UNIVERSITAS
TAURINENSIS

Founded in 1404

DEPARTMENT OF ECONOMICS AND
PUBLIC FINANCE "G. PRATO"
WORKING PAPER SERIES

POLITICAL ECONOMY OF DIRECTOR'S LAW: HOW SINCERE VOTERS DECIDE ON CASH AND IN-KIND REDISTRIBUTION IN A COSTLY POLITICAL FRAMEWORK



FRANCESCO SCERVINI

Working paper No. 8 - November 2009

Political economy of Director's law

How sincere voters decide on cash and in-kind redistribution in a costly political framework

Francesco Scervini*

November 30, 2009

Abstract

The amount of taxes and public expenditures seems to be uncorrelated to the level of market inequality in OECD countries. This empirical evidence is difficult to be rationalized in a standard median voter theorem setting, where individuals rationally choose their preferred redistribution scheme. This paper reconciles theory and evidence by introducing a source of political asymmetry, that is income inequality: assuming that political activity is costly, income distribution can be a determinant of political asymmetry, provided that some classes of individuals are not able to satisfy their political budget constraint.

The political framework consists of a bi-dimensional policy space where preferences over cash redistribution are monotonically decreasing with income, while those over in-kind redistribution depend on the middle class position, according to Director's law. The result is that the elected policy maker is increasingly biased toward rich classes of population as far as market income inequality increases.

JEL Classification: D31, D72, H42
Keywords: Income distribution, Redistribution, Political process, Publicly provided goods

*University of Torino. e-mail: francesco.scervini@unito.it
I sincerely thank Mario Ferrero, Giuseppe Bertola and Allan Drazen for the insightful suggestions. Of course, every mistake is my own exclusive responsibility.

1 Introduction

The link between inequality and redistribution is one of the most debated topics in the field of political economy. During the last decades, both theoretical and empirical evidence focused on this topic, finding different conclusions. The seminal paper by Meltzer and Richard (1981) links the steadily growing (in that period) size of governments to the increasing demand for redistribution from a relatively poorer median voter. In the early 90's a series of papers (Bertola, 1993; Alesina and Rodrik, 1994; Persson and Tabellini, 1994) extended the theoretical treatment to economic growth and attempted to empirically investigate the link between inequality and growth through politics and redistribution. Results of both theory and empirical evidence identify a positive relationship between inequality and redistribution, even if the role of the political framework is questioned by Alesina and Rodrik (1994).

Subsequently, empirical studies focused more directly on the role of politics in the transmission mechanism from inequality to redistribution, questioning not only that political framework plays a role in the determination of the level of redistribution, but also the positive relationship between inequality and redistribution (Perotti, 1996; Deininger and Squire, 1998; Milanovic, 2000; Scervini, 2009). These results gave an impulse to the branch of literature investigating the possible sources of the missing link between inequality and redistribution: Aghion and Bolton (1997) focused on trickle-down effects, Bénabou (1997) and Bénabou and Ok (2001) concentrated on the prospect of upward mobility, Besley and Coate (1998) and Acemoglu and Robinson (2000, 2005) referred to the asymmetry of *de facto* political power, while Saint-Paul and Verdier (1993) and Bourguignon and Verdier (2000) investigated the indivisibility of investments, following the direction of Galor and Zeira (1993).

The present paper merges two different issues. First of all, it investigates a possible source of political asymmetry that – unlike the previous contributions – depends on inequality. Second, it deals with one of the possible reasons why empirical literature detects the missing link between inequality and redistribution: the effect of general public expenditures on the middle class. Due to the shortage of reliable data, what empirical papers analyze is the effect of income inequality on the level of *cash* redistribution. However, it is possible that governments implement some redistribution through a channel different from direct cash transfers, that is other public expenditures. Public education, health care, infrastructures investments, housing, production and consumption subsidies must be considered as redistributive policies as far as they provide goods and services that individuals should otherwise buy in the private market.[1] Director's law, as stated by Stigler

[1]These policies can have also an additional redistributive effect, since they are a source

(1970), deals exactly with this problem, assessing that this kind of policies benefits the middle class much more than other individuals. In recent years, several empirical studies concentrated on this topic, finding that in-kind redistribution is indeed effective in reducing inequality.[2] However, the only recent cross-country study (Marical et al., 2006) confirms the result of a general inequality reduction, but it does not focus on the effects on the middle-class. Taking into account this issue should therefore reinforce the support in favor of the median voter theorem and the positive effective relationship between inequality and redistribution.

The paper is divided in two parts. The first deals with the asymmetry of political power: assuming that political activity is costly – inasmuch as political parties and institutions are necessary to enter the political competition – it is possible that some classes of individuals cannot afford to satisfy the budget constraint, either because they are too poor, or because they are too few individuals. The conclusion is that – under general and reasonable assumptions on the shape of income distribution – income inequality leads to a less complete political representation, since it may prevent some classes of individuals to constitute a political party.

In the second part of the model a bi-dimensional policy space is analyzed according to Director's law assumptions. The preferences over cash redistribution are assumed to be monotonically decreasing with income, according to all the models dealing with this issue. On the other side, given that in-kind redistribution is targeted to the middle class, the median voter should support this policy more than both richer and poorer individuals. The shape of preferences over in-kind redistribution, therefore, is non-monotonic with respect to income.

If we are ready to assume – and this is the only strong assumption of the model – that individuals vote sincerely for a party (if any) that implements at least one policy coincident to her own preferences, then the solution is that a higher income inequality leads to an amount of both cash and in-kind redistribution that is lower than what the median voter would choose.

The paper is organized as follows: after a summary of some motivating empirical evidence, section 2 focuses on Director's law and describes the political space and individual preferences on the policies, section 3 analyzes political costs and the effects of inequality on the symmetry of political representation and section 4 deals with two different assumptions regarding political framework and the respective solutions of the model. Section 5 concludes and the appendices include some analytical proves.

of income for the individuals employed by the government. However, if we assume that publicly provided private goods are produced with the same technology used by the private sector, then this effect is null.

[2]Among others, Aaberge and Langørgen (2006) for Norway, Baldini et al. (2007); D'Ambrosio and Gigliarano (2009) for Italy. Opposite, Sonedda and Turati (2005) question the reduction impact of in-kind transfers in Italy, using data on 2000.

1.1 Motivating evidence

The present section is devoted to sketch some rough empirical evidence that deals with the arguments treated above and to motivate some of the assumptions that I appeal to in the theoretical model.[3]

The first insightful result regards the link between inequality and the size of public sector. Table 1 shows that the correlations between the *market income* Gini index and the level of taxes and public expenditures are not statistically different from zero, while there is an intuitively strong positive correlation between taxes and expenditures. Moreover, the total tax burden is positively correlated to both cash and in-kind redistribution (table 2), the correlation with cash redistribution being much stronger.[4]

Variables	Ex-ante Gini index	Redistribution (%Δ Gini index)	Total public expenditures	Total tax revenues
Redistribution	0.264**	1.000		
	(0.011) [92]			
Total public	-0.050	0.782***	1.000	
expenditures	(0.714) [56]	(0.000) [56]		
Total tax	0.042	0.812***	0.896***	1.000
revenues	(0.697) [89]	(0.000) [89]	(0.000) [84]	

*** significant at 1%, ** significant at 5%, * significant at 10%
Observations in square brackets

Table 1: Tax revenues and public expenditures are uncorrelated to *ex-ante* inequality but correlated to redistribution in OECD countries.

Variables	Total tax revenues	Public exp. in-kind sectors	Public exp. social protection
Public expenditures,	0.484***	1.000	
in-kind sectors	(0.000) [81]		
Public expenditures,	0.825***	0.215*	1.000
social protection	(0.000) [81]	(0.052) [82]	

*** significant at 1%, ** significant at 5%, * significant at 10%
Observations in square brackets

Table 2: Total tax burden is correlated to both cash and in-kind redistribution in OECD countries.

Even if uncorrelated to market income inequality, the size of the public sector seems to be effective in reducing the level of inequality, as shown in

[3]Data on inequality, taxes and public expenditures come from the OECD dataset.

[4]By cash redistribution, I mean the social security transfers to individuals or households, while in-kind redistribution includes all the publicly provided private goods, such as education, health, housing. It is debatable whether public goods, such as defense, public order, environmental protection can be recorded as redistribution. In this analysis they are not included, however the results do not change under the alternative hypothesis.

table 1, where the level of redistribution is computed as the relative change between *ex-ante* and *ex-post* income inequality, and therefore takes only into account the amount of *cash* redistribution.

It is not surprising, but still interesting, that cash redistribution is uncorrelated to virtually any of the expenditure sectors (table 3) apart from social protection (that includes cash transfers and support to individuals and households). Housing, education, health – as predicted by Stigler (1970) – are not correlated to redistribution, meaning also that governments implementing more cash redistribution do not implement more in-kind redistribution.

Variables	Ex-ante Gini index	Redistribution (%Δ Gini index)
Total public expenditures	-0.050	0.782***
	(0.714)	(0.000)
General services	0.054	0.297**
	(0.691)	(0.026)
Defense	-0.072	-0.337**
	(0.599)	(0.011)
Public order and safety	0.423***	-0.402***
	(0.001)	(0.002)
Economic affairs	-0.207	0.257*
	(0.126)	(0.056)
Environment protection	0.029	0.003
	(0.830)	(0.983)
Housing	0.173	0.191
	(0.204)	(0.158)
Health	-0.142	0.084
	(0.295)	(0.538)
Recreation and culture	-0.073	0.719***
	(0.592)	(0.000)
Education	-0.204	0.202
	(0.131)	(0.136)
Social protection	0.007	0.796***
	(0.959)	(0.000)
Public expenditures, excluding social security	-0.020	0.290**
	(0.885)	(0.030)
Public expenditures, in-kind sectors	-0.175	0.249*
	(0.197)	(0.064)

*** significant at 1%, ** significant at 5%, * significant at 10%
56 observations

Table 3: Correlation between expenditure composition, inequality and redistribution in OECD countries.

Also the composition of taxation and its relationships to inequality and redistribution (table 4) deserve some comment. First of all, the total tax burden is uncorrelated to income inequality, suggesting that – even if it is very effective in fostering cash redistribution – the government size is not

Variables	Ex-ante Gini index	Redistribution (%Δ Gini index)
Total tax	0.042	0.812***
revenues	(0.697)	(0.000)
Income taxes	-0.325***	0.326***
	(0.002)	(0.002)
Social security	0.288***	0.428***
	(0.006)	(0.000)
Workforce payroll	0.010	0.395***
	(0.924)	(0.000)
Property taxes	-0.004	-0.468***
	(0.968)	(0.000)
Goods and services	0.084	0.620***
	(0.435)	(0.000)
Other taxes	0.420	0.151
	(0.000)	(0.158)

*** significant at 1%, ** significant at 5%, * significant at 10%
89 observations

Table 4: Correlations between tax composition, inequality and redistribution in OECD countries.

larger in more unequal countries. Moreover, the amount of income taxes is *negatively* correlated to the level of inequality. This apparent puzzling result can be driven by several factors, but it can be seen as a further evidence supporting the asymmetry of political power, where a rich-oriented policy maker sets an amount of redistribution negatively dependent on the income of rich individuals.[5] On the other side, this thesis is mitigated by the positive correlation between social security contributions and market income inequality.

A final minor remark refers to the unexpected sign of the correlation between expenditures in public order and safety and the level of both inequality and redistribution (table 3). Even if it is not possible to argue any causal relationship from the correlation index, one of the explanations for this result is that stronger social conflicts arise in more unequal societies, that are forced to devote significantly larger resources for public order whenever the level of inequality is higher and/or the level of redistribution is lower.

The empirical evidence presented in this section is very far from being supportive of any theoretical conclusion, nonetheless it can be considered as a broad picture of the relationship among several phenomena involved in this paper.

[5]For a more detailed treatment and empirical analysis on the topic, see Scervini (2009).

2 Policy space and individual preferences

The policy space consists of two dimensions, both related to fiscal policy-making. One is a simple textbook case of a linear income tax associated with a lump-sum cash transfer equal for all individuals. Also the second is a linear income tax, but its revenues are used to produce a set of private goods provided by the government. The characteristics of these private goods – according to Director's law (Stigler, 1970) – is that they benefit middle class individuals – or the median of the population – more than the others. The main argument is that these goods, such as housing support, education, health care, consumption subsidies, are not enjoyed by the poor classes of population because of the poverty trap mechanisms that prevent them from accessing many of these services. On the other side, rich individuals do not need the public sector to provide these goods, since they can access better quality services and therefore would prefer not to finance these kinds of goods. In the paper, I will refer for simplicity to the first policy as cash redistribution and to the second as in-kind redistribution.

Call τ and θ the two policy instruments related to cash and in-kind redistribution respectively, then the indirect utility function can be defined as

$$v(\tau,\theta) = (1-\tau-\theta)\, y_i + w\left(y_i^{\tau}(\tau)\right) + \alpha(y_i)\, z(g(\theta)) \tag{1}$$

where y_i is individual i (exogenous) market income, on which the tax rates τ and θ are applied, y_i^{τ} is the transfer received by individual i, g is the amount of in-kind redistribution provided by the government – equal for all the citizens – and $\alpha(y_i)$ is a weight that depends on the relative position of i in the income distribution according to the hypothesis of Director's law. Maximization of the function leads to the following first order conditions:

$$\begin{cases} \frac{\partial v}{\partial \tau} = & -y_i + \frac{\partial w}{\partial y_i^{\tau}} \frac{\partial y_i^{\tau}}{\partial \tau} = 0 \\ \frac{\partial v}{\partial \theta} = & -y_i + \alpha(y_i) \frac{\partial z}{\partial g} \frac{\partial g}{\partial \theta} = 0 \end{cases} \Rightarrow \begin{cases} \tau = & \tilde{w}(y_i) \\ \theta = & \tilde{z}(y_i)\, \tilde{\alpha}(y_i) \end{cases} \tag{2}$$

The additivity of the utility function implies that the level of the two income taxes preferred by an individual i are "independent" between each other. The preferences for the level of cash redistribution are unrelated to those for in-kind redistribution, even if both of them are intuitively related to individual market income.

In order to specify the implicit functions implied by the first order condition, we can look at the intuitive and desirable properties that should characterize the solution of indirect utility maximization:

- τ should be maximum at the lower extreme of the distribution, that is the poorest individual in the population.
- τ should be minimum for the richest individual in the population.

- τ should be strictly decreasing, meaning that individuals with different incomes have different preferences over the level of cash redistribution, and richer individuals prefer a lower level of redistribution.
- θ is maximum for the median individual in the population, according to Director's law.
- θ is symmetric with respect to the median, and strictly decreasing with the distance between individual income and the median voter.
- θ is minimum for the richest individual in the population. This assumption, together with symmetry, implies that the closer the median voter to the bottom of the distribution, the higher the level of θ preferred by the poorest individuals.
- $\tau + \theta \in [0, 1]$, $\tau \in [0, 1]$, $\theta \in [0, 1]$, ruling out negative tax rates or a total tax burden higher than the actual income.

The two simplest explicit policy preferences that satisfy all the above assumptions (see appendix 6.1 for the rigorous proofs) are the following:

$$\tau(y_i) = \frac{1}{2} - \frac{y_i}{2y_{max}} \tag{3}$$

$$\theta(y_i) = \frac{1}{2} - \frac{(y_i - y_{median})^2}{2(y_{max} - y_{median})^2} \tag{4}$$

and are represented in figure 1.

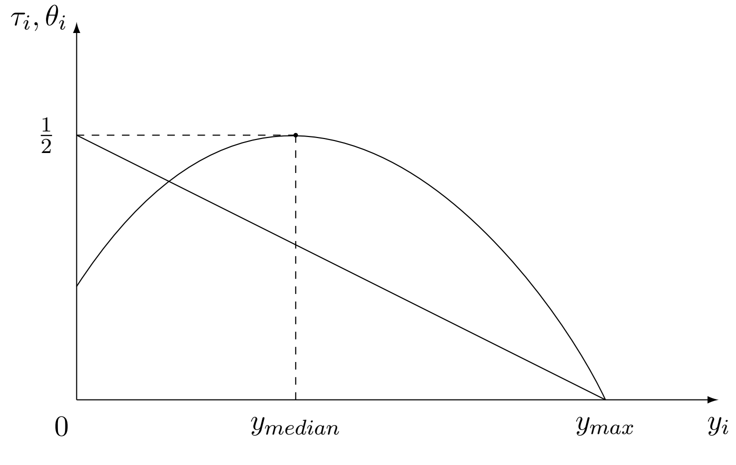


Figure 1: τ is linearly decreasing with income, while θ is symmetric with respect to the median and minimum for $y = y_{max}$

3 Inequality and political representation

One of the more innovative features of the model is that political activity is costly and therefore some classes of individuals are not able to participate in political competition because of the presence of some budget/liquidity constraint. First of all, I define a party as a group of individuals who share the same preferences over all political dimensions and get some utility from active participation in political life. In order to create a political party, these individuals need to join together and set up an organized institution that must be able to inform the voters about their policies, to interact with other institutions, mass media and so on. This set of activity implies a non-negligible cost – increasing with the party membership – that rests on the members of the party.

On the other side, every political-oriented individual is ready to spend a share of her income in order to create a political party and participate actively in the political competition.[6] For simplicity, we assume that the share of total income that every individual is ready to spend for political activity is constant.

Since – in this model – individual preferences are uniquely determined by personal income, it is straightforward to assume that all individuals with the same income have the same preferences over the policies, and therefore they join the same political group. Moreover, since each group is made up by individuals with the same preferences, they unanimously set their preferred policies as the target of their political activity.

The total amount of resources (R) available to a generic group, the costs (C) related to the political activity and the associated budget/liquidity constraint are the following:

$$R(y) = n_y y = f(y)\, y \tag{5}$$

$$C(y) = g(n_y) = g(f(y)) \tag{6}$$

$$f(y)\, y \geq g(f(y)) \tag{7}$$

where n_y is the number of individuals with income y and $f(y)$ the distribution of income, while $g(\cdot)$ is a generic cost function.

By solving the condition (7) with respect to y we retrieve the subset of groups that are effectively able to satisfy the budget constraint and create a political party. It is worthy to notice that there are two ways for satisfying the constraint: on the one side, if the mean income of a group is very low, the number of individuals must be high; on the other side, if it is the number of individual to be low, they must be very rich in order to satisfy the budget constraint. In general, however, the subset of parties depends on the shape of

[6]This innocuous assumption is common in the literature, see for instance Morelli (2004)

the income distribution and different scenarios can result from the process:[7]

$$y^* \in (0, y_{max}) \tag{8}$$
$$y^* \in (0, \overline{y}) \tag{9}$$
$$y^* \in \left(\underline{y}, y_{max}\right) \tag{10}$$
$$y^* \in \left(\underline{y}, \overline{y}\right) \tag{11}$$
$$y^* \in \left(0, \underline{y}\right) \cup (\overline{y}, y_{max}) \tag{12}$$
$$y^* \in \emptyset \tag{13}$$

where $\underline{y}$ and $\overline{y}$ are the two "marginal" groups, that is the poorest and richest group respectively able to satisfy the political budget constraint.

The possible solutions listed above represent the subset of groups that can afford to create a party and that – therefore – are the only possible policy makers. All other groups of individuals, being unable to satisfy the political budget constraint, are also prevented to be elected and to get the office.

What is really crucial for this model, however, is not only that political power is unevenly spread across the population, but also that the share of individuals able to compete in the political framework and their characteristics depend on the level of inequality. In order to investigate the effects of a change of the level of inequality on the subset of groups that can constitute a party, we need to compute the derivative(s) of the boundaries ($\underline{y}$ and $\overline{y}$) with respect to the parameter(s) of the distribution that determines the level of inequality. Of course, the procedure could become intractable if the shape of the distribution is complex, however I will go through the computations under a generic distribution $f(y)$, whose characteristics are described in the next section.

3.1 Political asymmetry

The present section is devoted to analytical treatment of sources of political asymmetry and of the effects of income inequality on the subset of groups that can effectively participate the political competition. First, I define the income distribution and the amount of resources and costs for political groups, then I analyze the political budget constraint and the groups that can satisfy it and, finally, I describe the effects of changes of income inequality on the ability of groups to constitute a political party. Strictly speaking, in what follows I name as "income inequality" an income distribution modification that is not mean-preserving, so that it could be more

[7] The following are the possible solutions under the assumption of a standard income distribution, "standard" meaning a single peaked distribution with median not higher than the mean. Of course, infinite cases are possible under other kinds of distributions. In the next sections a more detailed function specification will restrict the set of possible solutions, according to the characteristics of the income distribution and the cost function.

properly defined as "income asymmetry". However, on the one side, the results are not driven by the shift of the mean income, on the other, the two concepts are so intimately related that the cost of changing the common terminology is higher than the benefit. The reader should be aware of this remark in interpreting the following analysis.

Income y is distributed across population according to a single peaked function $f(y, \gamma)$ bounded between 0 and y_{max}, so that:

$$\int_0^{y_{max}} f(y, \gamma)\, dy = F(y, \gamma)|_0^{y_{max}} = 1 \tag{14}$$

where $f(0, \gamma) = f(y_{max}, \gamma) = 0$ and γ is a parameter denoting income inequality. It is possible to think of γ as a parameter such that an increase of inequality shifts an amount $\frac{\partial f(y,\gamma)}{\partial \gamma}$ of individuals from the right of the population to the left, across an arbitrary y^{**} where $\frac{\partial f(y,\gamma)}{\partial \gamma} = 0$. Among all the possible thresholds, an intuitive and convenient one is $y^{**} = y_{mode}$, meaning that an increase of inequality is represented in the model by a shift of individuals from the right to the left of the mode itself (see figure 2). Such a point is not only intuitive, but also convenient, since I am assuming that the mode is not affected by changes of income inequality, while the opposite is true for the mean and the median. The sign of the derivative is therefore positive (namely, an increase of population with income y) for all $y < y_{mode}$ and negative otherwise. Finally, we assume the two extremes of the population to be constant, so that the derivative is zero not only in $y = y_{mode}$, but also in $y = 0$ and $y = y_{max}$ (figure 3).

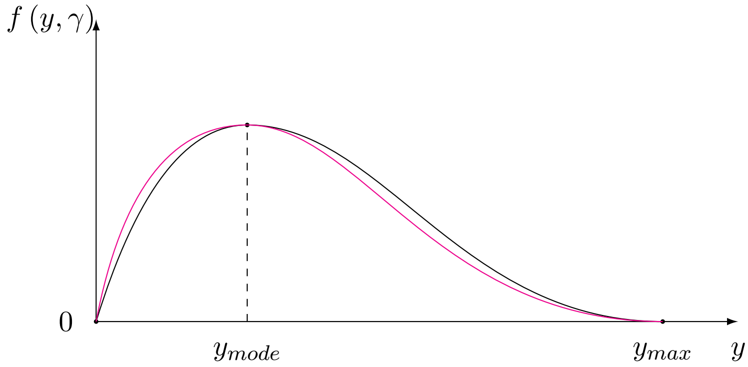


Figure 2: Assume a marginal increase of parameter γ. The effects on the income distribution are represented by the purple function, that is, an increase of individuals with income lower than the mode, and a decrease of richer ones. The mode is unchanged, as well as the minimum and the maximum values of income.

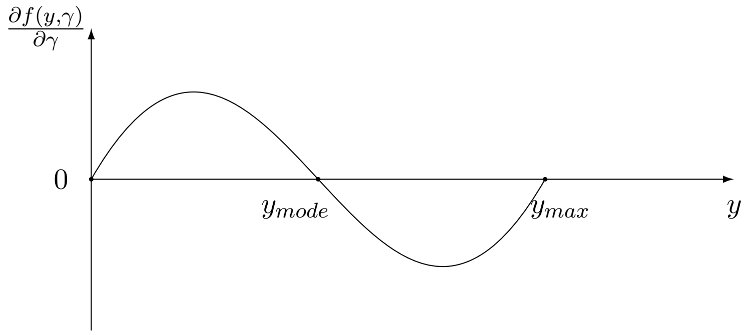


Figure 3: By definition of the function, a change of inequality shifts individuals $(f(y,\gamma))$ from the right to the left of a generic y, that is assumed to be $y = y_{mode}$, with $\frac{\partial f(y,\gamma)}{\partial \gamma} = 0$ if $y = 0$, $y = y_{mode}$, $y = y_{max}$.

With respect to income inequality, an increase of γ is associated to an increase of the Gini index. Indeed, if the share of "poor" increases with respect to share of "rich", the Lorenz curve shifts downward leading to an unambiguous increase of the Gini index. Figure 4 provides graphic intuition, while the proof is in appendix 6.2.

By definition of mode of a distribution, it is straightforward to notice that

$$\frac{\partial f(y,\gamma)}{\partial y} \begin{cases} > 0 & \text{if } y < y_{mode} \\ = 0 & \text{if } y = y_{mode} \\ < 0 & \text{if } y > y_{mode} \end{cases} \tag{15}$$

and

$$\frac{\partial f(y,\gamma)}{\partial \gamma} \begin{cases} > 0 & \text{if } \gamma < \gamma_{mode} \Rightarrow y < y_{mode} \\ = 0 & \text{if } \gamma = \gamma_{mode} \Rightarrow y = y_{mode} \\ < 0 & \text{if } \gamma > \gamma_{mode} \Rightarrow y > y_{mode} \end{cases} \tag{16}$$

The amount of resources available to every group in the population, as noticed in the previous section, is

$$R(y,\gamma) = f(y,\gamma)\, y \tag{17}$$

that is zero at the two extremes of the distribution. Indeed, by assumption, $f(0,\gamma) = f(y_{max},\gamma) = 0$. First derivative with respect to y is:

$$\frac{\partial R(y,\gamma)}{\partial y} = \frac{\partial f(y,\gamma)}{\partial y} y + f(y,\gamma) \begin{cases} > 0 & \text{if } \frac{\partial f(y,\gamma)}{\partial y} > -\frac{f(y,\gamma)}{y} \\ = 0 & \text{if } \frac{\partial f(y,\gamma)}{\partial y} = -\frac{f(y,\gamma)}{y} \\ < 0 & \text{if } \frac{\partial f(y,\gamma)}{\partial y} < -\frac{f(y,\gamma)}{y} \end{cases} \tag{18}$$

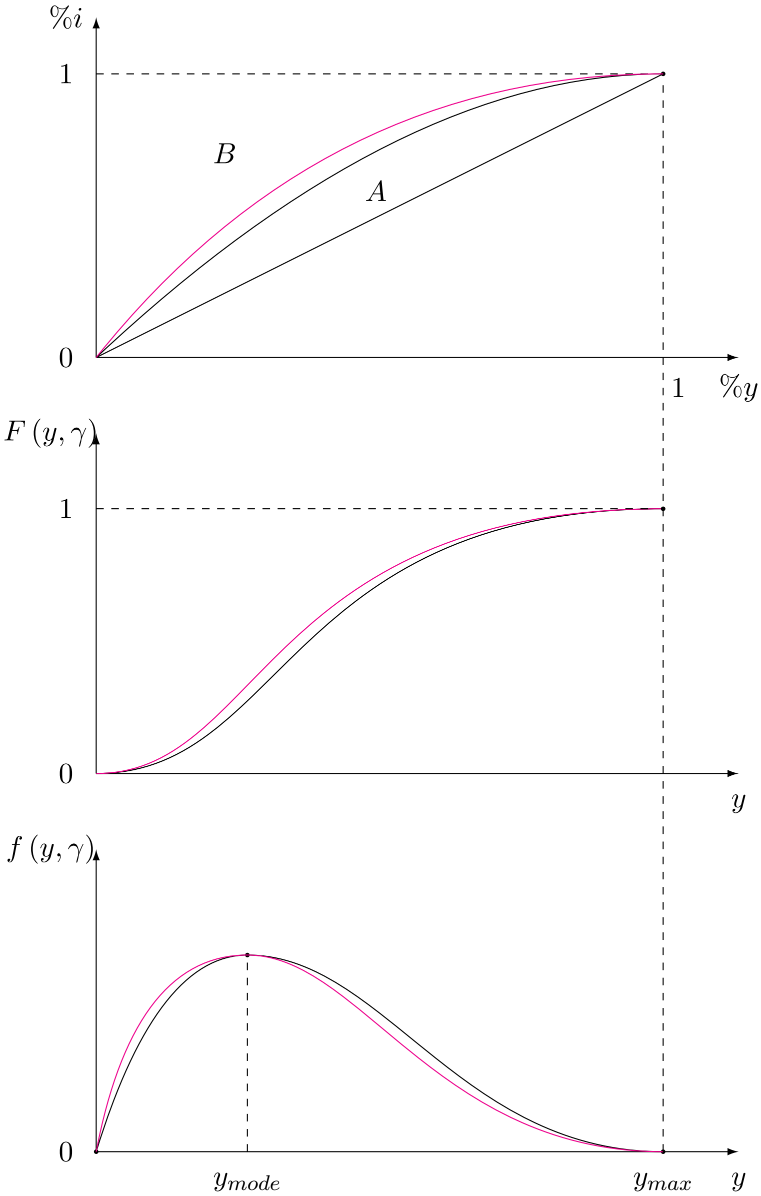


Figure 4: An increase of inequality shifts some individual from the right to the left of the mode, so that the cumulative distribution shifts leftward and so does also the Lorenz curve represented in the upper panel. Gini index ($\frac{A}{A+B}$) increases by definition.

Since the function is non-negative and takes value 0 at the extremes of the domain, the following optimality condition is also sufficient for the maximum:

$$\frac{\partial f(y,\gamma)}{\partial y} = -\frac{f(y,\gamma)}{y} \tag{19}$$

Combining conditions (19) and (15), it is straightforward to notice that the function is maximum for a generic $y^* > y_{mode}$, meaning – intuitively – that the maximum amount of resources is available to a group that is actually richer than the mode of the population.

On the other side, there are political costs, assumed to be positive and increasing with the group membership:

$$C(y,\gamma) = g(f(y,\gamma)) \tag{20}$$

with

$$g(0) = k > 0 \tag{21}$$

$$\frac{\partial g(f(y,\gamma))}{\partial f(y,\gamma)} > 0 \tag{22}$$

where k is a fixed cost, independent of the group membership. Total costs are therefore increasing up to y_{mode} and decreasing afterward, since:

$$\frac{\partial C(y,\gamma)}{\partial y} = \frac{\partial g(f(y,\gamma))}{\partial f(y,\gamma)}\frac{\partial f(y,\gamma)}{\partial y}\begin{cases} > 0 & \text{if } \frac{\partial f(y,\gamma)}{\partial y} > 0 \\ = 0 & \text{if } \frac{\partial f(y,\gamma)}{\partial y} = 0 \\ < 0 & \text{if } \frac{\partial f(y,\gamma)}{\partial y} < 0 \end{cases} \tag{23}$$

Since $C(y,\gamma)$ is a positive monotonic transformation of $f(y,\gamma)$, the first order conditions are also sufficient and $y_{mode} = \arg\max C(y,\gamma)$

Comparing the two points of maximum for costs and resources, it is easy to notice that there is an interval between y_{mode} and the generic y^* in which political costs are decreasing, while resources are increasing. This suggests that the maximum of the budget function is at some point higher than y^*. In order to specify the shape and the characteristics of the function, we can now analyze the shape of the groups budget constraint, that is simply the difference between groups' resources and political costs:

$$B(y,\gamma) = R(y,\gamma) - C(y,\gamma) = f(y,\gamma)\,y - g(f(y,\gamma)) \tag{24}$$

Given the properties above, the function is always negative at the two extremes, since:

$$B(0,\gamma) = R(0,\gamma) - C(0,\gamma) = 0 - g(0) = -k < 0 \tag{25}$$

First order condition is:

$$\frac{\partial B(y,\gamma)}{\partial y} = \frac{\partial f(y,\gamma)}{\partial y}y + f(y,\gamma) - \frac{\partial g(f(y,\gamma))}{\partial f(y,\gamma)}\frac{\partial f(y,\gamma)}{\partial y} = 0 \tag{26}$$

$$\Rightarrow \frac{\partial f(y,\gamma)}{\partial y}\left[y - \frac{\partial g(f(y,\gamma))}{\partial f(y,\gamma)}\right] = -f(y,\gamma) \tag{27}$$

Condition (27) is satisfied if:

$$\frac{\partial f(y,\gamma)}{\partial y} = -\frac{f(y,\gamma)}{y - \frac{\partial g(f(y,\gamma))}{\partial f(y,\gamma)}} \begin{cases} > 0 & \text{if } y < y_{mode} \\ < 0 & \text{if } y > y_{mode} \end{cases} \tag{28}$$

$$\text{so that } \begin{cases} \frac{\partial g(f(y,\gamma))}{\partial f(y,\gamma)} > y \Leftrightarrow y < y_{mode} \\ \frac{\partial g(f(y,\gamma))}{\partial f(y,\gamma)} < y \Leftrightarrow y > y_{mode} \end{cases} \tag{29}$$

A generic $\hat{y} \in (0, y_{max})$ that satisfies the first order conditions can be either a maximum or a minimum. However, since we know that in y_{mode} the derivative of the constraint function with respect to y is positive[8] then if $\hat{y} > y_{mode}$, it is a maximum, otherwise it is a minimum at $R(y,\gamma) - C(y,\gamma) < -k < 0$. Figure 5 shows the income distribution and all the functions described in this section.

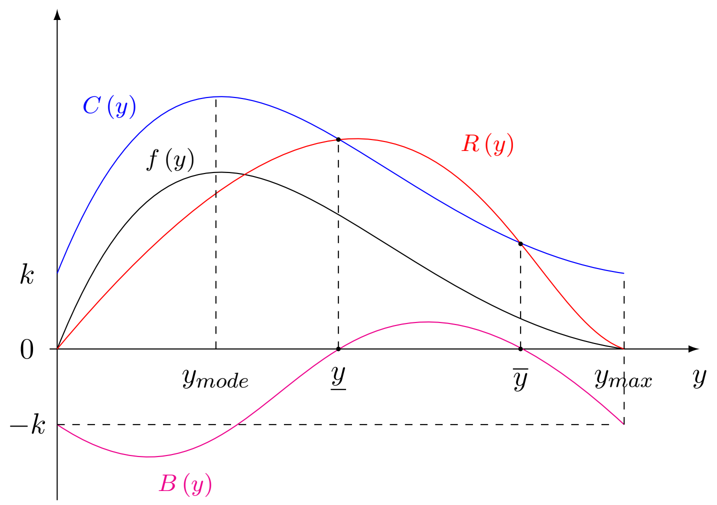


Figure 5: The cost function $C(y)$ is a positive transformation of the distribution $f(y)$ (assumed in the picture to be simply $C(y) = k + f(y)$), while the budget function $B(y)$ is the difference between resources $R(y)$ and costs $C(y)$.

The most insightful result is that there are two thresholds, $\underline{y}$ and $\overline{y}$ such that $R(y,\gamma) = C(y,\gamma)$. Indeed, the budget function is negative at the two extremes. In principle, it is possible to think of a function such that $R(y,\gamma) < C(y,\gamma)$ also at the maximum, but this would imply that no groups

[8] Indeed, the derivative of $R(y,\gamma)$ is positive, while that of $C(y,\gamma)$ is zero.

have resources enough to set-up a political party. Such condition could be true for poor countries where, independently of the level of inequality, the income is so low (or, the political costs are so high) that individuals are virtually never able to participate the political framework and dictatorships arise. The position of the two thresholds is also interesting. The upper one, $\overline{y}$ is always greater than $\hat{y}$, that in turn implies $\overline{y} > y_{mode}$. The lower threshold, instead, is lower than $\hat{y}$, but nothing can be said about its relative position with respect to y_{mode}.

Recall that the crucial question of the model is how the level of inequality affects the political framework. In order to answer this question, we should analyze the derivatives of both costs and resources with respect to γ at the two thresholds, that is where $R(y,\gamma) = C(y,\gamma)$. First of all, we analyze the position of the thresholds. As noticed previously, the upper threshold is always higher than $\hat{y}$, while the second is always lower, and in turn it can be higher or lower than the mode of the population, that is:

$$y_{mode} < \hat{y} < \overline{y} \tag{30}$$

$$y_{mode} < \underline{y} < \hat{y} \vee \underline{y} < y_{mode} < \hat{y} \tag{31}$$

If we derive the budget constraint with respect to the inequality parameter γ, we obtain:

$$\frac{\partial B}{\partial \gamma} = \frac{\partial f(y,\gamma)}{\partial \gamma}\left[y - \frac{\partial g(f(y,\gamma))}{\partial f(y,\gamma)}\right] \tag{32}$$

whose sign at the thresholds depends crucially on the relative position of the thresholds with respect to the mode:

$$\text{If } y_{mode} < \hat{y} < \overline{y} \tag{33}$$

$$\Rightarrow \frac{\partial f(y,\gamma)}{\partial \gamma} < 0, \left[y - \frac{\partial g(f(y,\gamma))}{\partial f(y,\gamma)}\right] > 0 \tag{34}$$

$$\Rightarrow \frac{\partial B(\overline{y},\gamma)}{\partial \gamma} < 0 \tag{35}$$

$$\text{If } y_{mode} < \underline{y} < \hat{y} \tag{36}$$

$$\Rightarrow \frac{\partial f(y,\gamma)}{\partial \gamma} < 0, \left[y - \frac{\partial g(f(y,\gamma))}{\partial f(y,\gamma)}\right] > 0 \tag{37}$$

$$\Rightarrow \frac{\partial B(\underline{y},\gamma)}{\partial \gamma} < 0 \tag{38}$$

$$\text{If } \underline{y} < y_{mode} < \hat{y} \tag{39}$$

$$\Rightarrow \frac{\partial f(y,\gamma)}{\partial \gamma} > 0, \left[y - \frac{\partial g(f(y,\gamma))}{\partial f(y,\gamma)}\right] < 0 \tag{40}$$

$$\Rightarrow \frac{\partial B(\underline{y},\gamma)}{\partial \gamma} < 0 \tag{41}$$

What emerges from these results is that the two thresholds converge to $\hat{y}$ for steadily increasing inequality. This means that an increase of inequality is associated to higher costs and less resources for both rich and poor groups.[9] Therefore, we can state that there are two thresholds, $\underline{y}(\gamma)$ and $\overline{y}(\gamma)$, such that:

$$\frac{\partial \overline{y}(\gamma)}{\partial \gamma} < 0 \tag{42}$$

$$\frac{\partial \underline{y}(\gamma)}{\partial \gamma} > 0 \tag{43}$$

A further insightful consideration is that, in general, an increase of income inequality can be seen as a transfer of income from a poorer individual to a richer one, no matter where they are in the distribution. In the particular case considered here, an increase of the parameter γ consists of a transfer from *all* the individuals poorer than $\hat{y}$ to *all* individuals richer than that threshold. While the mode is unaffected by definition of γ, the effects of such a change on the median of the distribution is always negative. Indeed, by definition the median of the population decreases whenever – as assumed in this model – some individual of the population "moves" from the right to the left of the median. Finally, intuition suggests that also the mean of the population is affected by a change of the income distribution, decreasing as inequality increases so that:

$$\frac{\partial y_{mode}(\gamma)}{\partial \gamma} = 0 \tag{44}$$

$$\frac{\partial y_{median}(\gamma)}{\partial \gamma} < 0 \tag{45}$$

$$\frac{\partial y_{mean}(\gamma)}{\partial \gamma} < 0 \tag{46}$$

4 Electoral process

In the previous sections I have described the process of party formation and its relationship to the income distribution. The topic of the present section is to model an electoral procedure that links individual preferences to voting behavior, considering two different settings: a simple one-policy scenario and a more realistic bi-dimensional framework, that allows to investigate the central relationship between cash and in-kind redistribution.

[9] In fact, the result is partially driven by the choice of the threshold y^{**} at which $\frac{\partial f(y,\gamma)}{\partial \gamma} = 0$. If we assumed it at a generic income y^{**} higher than the mode, there would be a small region of convergence to the mode, for groups $y \in [y_{mode}, y^{**}]$, where resources increase more than costs, making it easier for those groups to set up a party. However, the possible convergence does not modify their relationship with the median voter position and does not affect the main insights of the model.

The main assumption relative to the voting system is that an individual casts her ballot for the party (if any) whose policies coincides with her preferences, otherwise she abstains. This assumption can be justified both theoretically and empirically (see the good review by Geys (2006)) and it is qualitatively robust to modifications of the main assumption. The ideal framework would be that individuals not perfectly represented by any party decide to vote for the closest one, provided it is not too far from her position in the policy space. What I assume is that the definition of "too far" is very strict, and all the parties with non coincident policies are considered "too far" by electors.[10] This assumption can seem very restrictive, but it does not change qualitatively the results of the model, with the advantage of a substantial simplification.

4.1 Single policy space

Assume that the policy space consists of a single redistributive policy τ, as described in section 2. By setting $\theta = 0$, utility maximization problem shrinks to the simple disposable income maximization with respect to the linear income tax, leading to condition (3), reported for convenience hereafter:

$$\tau_i^* = \frac{1}{2} - \frac{y_i}{2y_{max}} \tag{47}$$

On the other side, in a single policy space the voting rule assumed above in this section can be formally rationalized as follows:

$$v_i(y_i) = \begin{cases} y_i & \text{if } y_i \in [\underline{y}, \overline{y}] \\ \emptyset & \text{if } y_i \notin [\underline{y}, \overline{y}] \end{cases} \tag{48}$$

The winning party should be the one receiving more support, that is the one that represents more individuals. Opposite to the predictions of a median-voter theorem setting, in the present model parties are "stick" to the position preferred by their members and cannot shift along the policy line. The winning party is therefore either the one at the mode of the distribution or the closest to it, if the budget constraint prevents the group at the mode from setting up a party. Since both the subset of political parties and the thresholds depend on the level of inequality, also the position and the characteristics of the winning party does depend on it.

The number of individuals who decide to vote is given by the sum of people represented by a political party, that is, more formally, the subset of individuals with income included in the bracket $[\underline{y}, \overline{y}]$:

$$\int_{\underline{y}}^{\overline{y}} f(y)\, dy \tag{49}$$

[10] A further element supporting this condition is the usual decrease of turnout between first and second round in two ballots electoral systems. If individuals always preferred to vote for the "closest" party, we should observe no differences at all.

Previously, we found that the optimal tax rate for a generic individual i depends on the level of income she earns. However, we are interested in understanding how the policy preferences of the parties change after an increase of the level of income inequality. It is possible to investigate this issue by looking at the following relationship:

$$\frac{\partial \tau_i \left(y_i \left(\gamma\right)\right)}{\partial \gamma} = \frac{\partial \tau_i \left(y_i \left(\gamma\right)\right)}{\partial y_i \left(\gamma\right)} \frac{\partial y_i \left(\gamma\right)}{\partial \gamma} = -\frac{1}{2 y_{max}} \frac{\partial y_i \left(\gamma\right)}{\partial \gamma} \tag{50}$$

computed at "interesting" levels of income, such as y_{median}, $\overline{y}$ and $\underline{y}$.

In particular, in this simple framework individuals vote sincerely according only to one policy dimension, so that every party i gets an amount of votes exactly equal to the number of individuals with income y_i. The resulting winning party is therefore the one for which:

$$f\left(y_i\right) > f\left(y_j\right) \forall j \neq i, y_i \in \left[\underline{y}, \overline{y}\right] \tag{51}$$

that is, either the mode of the population – if it is included in the interval $\left[\underline{y}, \overline{y}\right]$ – or the group closest to it, that is some group k with income $y_k = \underline{y} > y_{mode}$.

How does the level of redistribution change in response to an increase of income inequality? What happens is that – since the increase of the group membership is lower than the increase of the associated political costs – an increase of income inequality makes it more difficult for the marginal group to constitute a party, so that the threshold $\underline{y}$ shifts toward the rich tail of the distribution. From an analytical perspective, it can be represented as follows:

$$\frac{\partial \tau_i \left(\underline{y} \left(\gamma\right)\right)}{\partial \gamma} = -\frac{1}{2 y_{max}} \frac{\partial \underline{y} \left(\gamma\right)}{\partial \gamma} < 0 \tag{52}$$

The effects on the median voter preferences are opposite, namely, an increase of income inequality makes the median voter poorer, moving her preferences toward a higher level of redistribution, since

$$\frac{\partial \tau_i \left(y_{median} \left(\gamma\right)\right)}{\partial \gamma} = -\frac{1}{2 y_{max}} \frac{\partial y_{median} \left(\gamma\right)}{\partial \gamma} > 0 \tag{53}$$

Indeed, the distance between the poorest party and the median voter increases if the latter is poorer than the former, otherwise it decreases. However, for steadily increasing inequality, the median ends up to be always lower than the poorest party, and therefore the distance between the policy maker and the median voter always increases for a level of inequality higher than a given threshold.

An analogous argument holds if the mode of the distribution is included in the subset of the constituted parties. In this case, the party at the mode of population is the policy maker, and the preferences of the median voter are

	Case (1)	Winner (2)	τ_{winner} (3)	τ_{median} (4)	$\Delta\tau$ (5)
1	$y_{mode} < y_{median} < \underline{y}$	$\underline{y}$	$\downarrow$	$\uparrow$	$\uparrow$
2	$y_{mode} < \underline{y} < y_{median}$	$\underline{y}$	$\downarrow$	$\uparrow$	$\downarrow$
3	$y_{median} < y_{mode} < \underline{y}$	$\underline{y}$	$\downarrow$	$\uparrow$	$\uparrow$
4	$y_{median} < \underline{y} < y_{mode}$	y_{mode}	$\leftrightarrow$	$\uparrow$	$\uparrow$
5	$\underline{y} < y_{mode} < y_{median}$	y_{mode}	$\leftrightarrow$	$\uparrow$	$\downarrow$
6	$\underline{y} < y_{median} < y_{mode}$	y_{mode}	$\leftrightarrow$	$\uparrow$	$\uparrow$

Table 5: Effects of an increase of inequality on the preferred amount of cash redistribution.

decreasing with income inequality. Again, for inequality higher than a certain threshold, the median voter is poorer than the mode of the population and the distance between the preferred policies increases with inequality.

Table 5 summarizes all the possible cases, according to the relative position of median, mode and lower threshold. First of all, it is possible to notice that the level of redistribution set by the policy maker (that is, the winner party) is lower than that preferred by the median voter in four cases over six (in particular, rows 1, 3, 4, 6 in table 5). Intuitively enough, the two cases in which the actual redistribution is *higher* than the preferences of the median voter are those in which both the mode and the lower threshold are lower than the median, a situation in which political power is very spread across individuals, either because of a low level of inequality (if $\underline{y} < y_{mode} < y_{median}$) or because of a low level of political costs ($y_{mode} < \underline{y} < y_{median}$). Opposite, if the policy maker is richer than the median voter, she will set an amount of redistribution lower than the one preferred by the median voter. It is possible to imagine a sort of multiple equilibrium: if the level of "initial" asymmetry is low, redistributive policy is actually stronger than that preferred by the median voter, and an equalizing equilibrium arises in which redistribution fosters political participation. On the other side, if the level of asymmetry is high, the policy is less redistributive and asymmetry increases, leading to a high inequality - low redistribution equilibrium.

How does an exogenous change of inequality influence the distance between the policy maker and median voter preferences? The effects of inequality on the median of the population are always the same: since the median becomes poorer, the median voter always prefers a higher amount of redistribution (column 4). If the lower threshold is higher than the mode of the population, it is the winner party and its preferences over redistribution decreases with inequality. As a consequence, if it is richer than the median voter (rows 1 and 3) the distance to the median voter increases, otherwise it decreases (row 2). On the other side, if the winner party is the mode, then its preferences are unchanged and the distance to the median voter depends again on their relative position.

It is crucial to stress that steadily increasing inequality always leads to the case in which the median is lower than the mode, so that the distance between the median voter and the policy maker is always increasing in highly unequal systems.

4.2 Bi-dimensional policy space

Consider now the more realistic framework with a bi-dimensional policy space, where the voters cast their ballots according to both policy instruments τ and θ. The voting rule is the same as that described in the single-policy space. Analogously, an individual votes for the party with coincident preferences. However, a necessary modification is required in a two policy space: if such a party – with preferences coincident on both policy instruments – does not exist, then the individual casts her ballot for a party with policy coincidence for one dimension. If this party does not exist either, then the individual abstains.

A careful reader could argue that such a voting behavior is unrealistic, at least for voters very close to the poorest marginal parties, $\underline{y}$, that are supposed to vote for a party that implements a very different policy on one dimension (τ) but the same policy on the other (θ), instead of voting for $\underline{y}$ that implements very similar – even if not coincident – policies on both dimensions. The reader can be right, but this seemingly unrealistic assumption can be justified by several considerations: first, it is very difficult to compare the marginal utilities of the two policies, and any assumption on this is equally arbitrary. Second, as remarked above, if we do not assume "perfectly sincere" voters, we should assume anyway that there is an arbitrary policy distance beyond which voters abstain. Finally, even if we tried to model such a distance, then the voters behavior would depend on the specific shape of the policy functions, $\tau(y_i)$ and $\theta(y_i)$.

Keeping these caveats in mind, the following voting function is the bi-dimensional analogous to function 48 for the single-policy space:

$$v_i(y_i) = \begin{cases} y_j & \text{if } \tau(y_j) = \tau(y_i), \theta(y_j) = \theta(y_i), y_j \in \left[\underline{y}, \overline{y}\right] \\ y_k & \text{if } \theta(y_k) = \theta(y_i), y_j \notin \left[\underline{y}, \overline{y}\right], y_k \in \left[\underline{y}, \overline{y}\right] \\ \emptyset & \text{if } \tau(y_l) \neq \tau(y_i), \theta(y_l) \neq \theta(y_i), \forall y_l \in \left[\underline{y}, \overline{y}\right] \end{cases} \tag{54}$$

The identity of the first group of voters is straightforward: all individuals whose group is able to constitute a party vote for it, since they share with it the preference over both policies. The second group is made up by all individuals who do not find perfect correspondence with any party, but who share the preferences over θ with the party "symmetric" to their group with respect to the median. Indeed, consider a voter with $y_i \in \left[0, \underline{y}\right]$ whose group is not able to constitute a party. It is straightforward (figure 6) that there are no other groups with the same preferences over τ, since the only solution for

$$\tau(y_l) = \tau(y_i) \tag{55}$$

$$\frac{1}{2} - \frac{y_l}{2y_{max}} = \frac{1}{2} - \frac{y_i}{2y_{max}} \tag{56}$$

is $y_l = y_i \notin [\underline{y}, \overline{y}]$. On the other side, it is possible to find a group with the same preferences over θ, since

$$\theta(y_l) = \theta(y_i) \tag{57}$$

$$\frac{1}{2} - \frac{(y_l - y_{median})^2}{2(y_{max} - y_{median})^2} = \frac{1}{2} - \frac{(y_i - y_{median})^2}{2(y_{max} - y_{median})^2} \tag{58}$$

has two solutions: $y_l = y_i \notin [\underline{y}, \overline{y}]$ and $y_l = 2y_{median} - y_i$. There is, therefore, a group of individuals with income $y_l = 2y_{median} - y_i$ with the same preferences on θ and different preferences on τ. If the group y_l is able to constitute a party, that is $y_l \in [\underline{y}, \overline{y}]$, then individuals with income y_i will cast their ballot for that party, provided of course that $y_i \notin [\underline{y}, \overline{y}]$ (that is, there is are no parties with the same preferences also on τ).

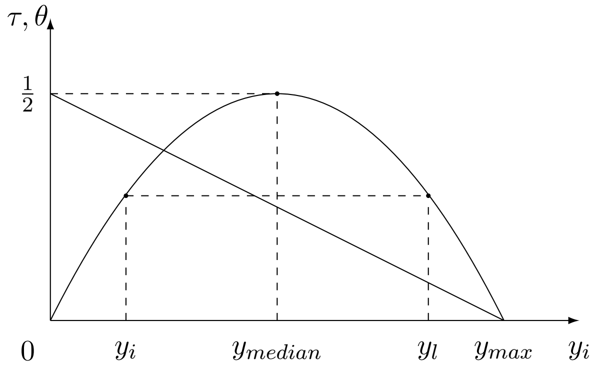


Figure 6: The two policies are represented on the vertical axis. The amount of cash redistribution τ is linearly decreasing with income, while in-kind redistribution is increasing up to the median (in the picture, we assume a symmetric income distribution) and decreasing afterward. Party y_l has preferences on θ coincident with those of group y_i, while different preferences on τ.

The third group of individuals – those with income $y_i \notin [\underline{y}, \overline{y}]$ whose symmetric group is $y_l \notin [\underline{y}, \overline{y}]$ – cannot find any party with preferences correspondent to their own ones and all its members abstain.

Focus now on the effects of a change of inequality on the groups' preferences with respect to the two policy instruments. Regarding τ, the same argument made in the previous section apply in this case, while we need to analyze hereafter the effects on the in-kind redistribution policy instrument, θ. In general, preferences of a generic group y_i change according to the following derivative (60):

$$\theta_i\left(y_i\left(\gamma\right), y_{median}\right) = \frac{1}{2} - \frac{\left[y_i - y_{median}\right]^2}{2\left[y_{max} - y_{median}\right]^2} \tag{59}$$

$$\frac{\partial\theta}{\partial\gamma} = \frac{\partial\theta}{\partial y_i}\frac{\partial y_i}{\partial\gamma} + \frac{\partial\theta}{\partial y_{median}}\frac{\partial y_{median}}{\partial\gamma} = \tag{60}$$

$$= \frac{y_i\left(\gamma\right) - y_{median}\left(\gamma\right)}{\left(y_{max}\left(\gamma\right) - y_{median}\left(\gamma\right)\right)^2}\left(\frac{y_{max} - y_i\left(\gamma\right)}{y_{max} - y_{median}\left(\gamma\right)}\frac{\partial y_{median}\left(\gamma\right)}{\partial\gamma} - \frac{\partial y_i\left(\gamma\right)}{\partial\gamma}\right) \tag{61}$$

The function is clearly invariant at the median of the population, since the median group prefers the maximum amount of in-kind redistribution by assumption. The preferences over θ of the poor marginal party, $\underline{y}$, are increasing with inequality if it is poorer than the median, decreasing otherwise. Intuitively, when inequality increases, the median shifts leftward (namely, it becomes poorer) and the marginal party shift rightward (it becomes richer), so that they get closer if the latter is poorer than the former, farther otherwise. Therefore, in case of a low level of inequality, the difference narrows, otherwise it spreads.

A similar argument can be made with respect to the mode of the distribution. Also in this case, if the mode is poorer than the median, then their preferences converge, in the opposite case they diverge. However, in both cases, it must be remarked that a steadily increasing inequality leads the preferences of the mode of the population and of the poor marginal party to become less redistributive oriented with respect to the median voter position.

Opposite to these cases, when we refer to the rich marginal party, $\overline{y}$, the results are always ambiguous. Indeed, the effects of increasing inequality are the same: both y_{median} and $\overline{y}$ shift leftward, becoming poorer, and the total effect on θ is therefore ambiguous without further specifications.

4.3 Winning party

With respect to the case of a single policy space, the present framework is less straightforward. The winning party is no more coincident (or the closest) to the mode of the population, since – in the present case – there is a subset of individuals who vote for a party with preferences only partially coincident with their own ones, and the sum of the two groups can be higher than the mode. In general, the population is partitioned in three types of individuals: those who vote for the party they belong, with perfect coincidence for τ and θ, those who cannot constitute a party and vote for the party symmetric to their group with respect to the median, since it represents their preferences on the instrument θ, and those who cannot set-up a party and whose symmetric group – that shares with them the preferences over θ – is neither able to constitute a party, and therefore abstain (figure 7).

There are several possible scenarios, according to the relative position of the marginal parties with respect to the median and the mode of the population. Table 6 summarizes the results, while the complete analytical treatment is reported in appendix 6.3. In what follows I will discuss the main economic implications on the level of cash and in-kind redistribution.

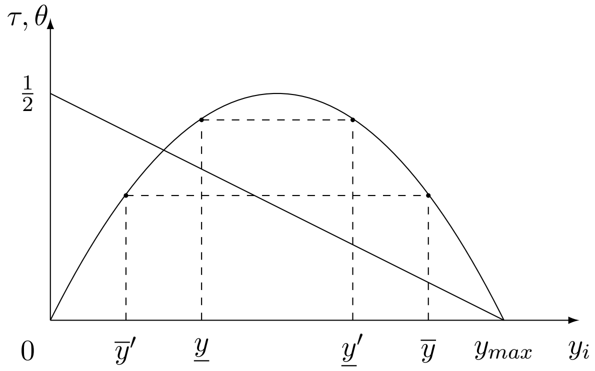


Figure 7: Individuals with income $y_i \in [0, \overline{y}']$ and $y_i \in [\overline{y}, y_{max}]$ abstain, those with income $y_i \in \left[\overline{y}', \underline{y}\right]$ vote for the corresponding symmetric party $y_k \in \left[\underline{y}', \overline{y}\right]$, while individuals $y_i \in \left[\underline{y}, \overline{y}\right]$ vote for their party y_i, where $\overline{y}' = 2y_{median} - \overline{y}$ and $\underline{y}' = 2y_{median} - \underline{y}$.

First of all, it should be noticed that the policies preferred by the policy maker (relative to either dimensions) are always less redistributive than those preferred by the median voter, apart from the particular case in row 5 of table 6: if the distribution is very equal and the political power is very evenly spread across individuals (i.e., low inequality and low political costs) then it is possible that $\underline{y} < y_{mode} < y_{median}$ and $f(y_{mode}) > f\left(\underline{y}\right) + f\left(\underline{y}'\right)$. In this case, the policies implemented are actually more redistributive than those preferred by the median voter, and therefore a kind of redistributive equilibrium can realize, leading to an equalizing redistribution that fosters political participation. In all the other cases, however, the amount of redistribution is always lower than that preferred by the median voter with respect to both political dimensions.

On the other side, the effects of an exogenous change of inequality may be different. The common conclusion is that if political asymmetry is high (namely, $y_{mode} < \underline{y}$), the distance between the preferences of the median voters and those of the policy makers increases toward a less redistributive policy, while if the level of asymmetry is lower (i.e. $\underline{y} < y_{mode}$) then the effects are ambiguous, and very small in magnitude. This means that redistribution is anyway lower than what preferred by the median voter, but the difference does not increase.

With respect to the single policy space, in the bi-dimensional case the levels of redistribution are more often lower than those preferred by the median voter. Moreover, while in the previous case it is possible that an exogenous increase of inequality narrows the difference between the policy maker and the median voter, this is never true in the two-policies space. The key difference is the fact that poor individuals can vote for rich parties, sharing with them the same preferences over the level of in-kind redistribution.

Finally, we see that changes in inequality have always the same qualitative effects on the two policies in case of "high political asymmetries", that is if $y_{median} < \underline{y}$, while the effects on cash redistribution are orthogonal to those on in-kind redistribution if the political asymmetries are low. Indeed, table 6 shows that in the former case (even lines) τ_{winner} and θ_{winner} are always decreasing, while in the latter case (odd lines) τ_{winner} is increasing, while θ_{winner} is ambiguous, and however much smaller in absolute values, since the two effects (the shift of the median and the shift of the marginal party) act in opposite directions.

The positive and significant correlation between cash and in-kind redistribution in table 2 could suggest that the policy makers in OECD countries belong to the first category of parties, with income higher than the median voter, being aware of the rough nature of the empirical evidence.

5 Conclusion

The first part of the paper investigates the effects of costs associated to political activity on the ability of groups to effectively participate the political competition. With respect to most of the previous literature, this paper does not *assume* asymmetry of political power, but it attempts to investigate one possible explanation of such asymmetry.[11] An insightful result is that – under very general assumptions on the shape of the distribution and on the structure of political costs – increasing inequality tends to narrow

[11] Many very insightful papers seem to deviate from the assumption of universal *de iure* political power, assuming – for instance – that only educated individuals have political power (Bourguignon and Verdier, 2000). Even if there is some evidence that political power is unevenly spread across individuals, such an assumption can be considered extreme in a political system where universal franchise is guaranteed.

	Case	Winner	τ_{winner}	τ_{median}	$\Delta\tau$	θ_{winner}	θ_{median}	$\Delta\theta$
1	$y_{mode} < \overline{y}' < \underline{y} < y_{median} < \underline{y}' < \overline{y}$	$[\underline{y}', \overline{y}]$	$\uparrow$	$\uparrow$	?	?	$\leftrightarrow$	?
2	$y_{mode} < \overline{y}' < \underline{y}' < y_{median} < \underline{y} < \overline{y}$	$[\underline{y}, \overline{y}]$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\leftrightarrow$	$\uparrow$
3	$\overline{y}' < y_{mode} < \underline{y} < y_{median} < \underline{y}' < \overline{y}$	$[\underline{y}', y'_{mode}]$	$\uparrow$	$\uparrow$	?	?	$\leftrightarrow$	?
4	$\overline{y}' < y_{mode} < \underline{y}' < y_{median} < \underline{y} < \overline{y}$	$[\underline{y}, y'_{mode}]$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\leftrightarrow$	$\uparrow$
5	$\overline{y}' < \underline{y} < y_{mode} < y_{median} < \underline{y}' < \overline{y}$	y_{mode} or $\underline{y}'$	$\leftrightarrow$ or $\uparrow$	$\uparrow$	$\downarrow$ or ?	$\uparrow$ or ?	$\leftrightarrow$	$\downarrow$ or ?
6	$\overline{y}' < \underline{y}' < y_{mode} < y_{median} < \underline{y} < \overline{y}$	$\underline{y}$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\leftrightarrow$	$\uparrow$
7	$\overline{y}' < \underline{y} < y_{median} < y_{mode} < \underline{y}' < \overline{y}$	y_{mode} or $\underline{y}'$	$\leftrightarrow$ or $\uparrow$	$\uparrow$	$\uparrow$ or ?	$\downarrow$ or ?	$\leftrightarrow$	$\uparrow$ or ?
8	$\overline{y}' < \underline{y}' < y_{median} < y_{mode} < \underline{y} < \overline{y}$	$\underline{y}$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\leftrightarrow$	$\uparrow$
9	$\overline{y}' < \underline{y} < y_{median} < \underline{y}' < y_{mode} < \overline{y}$	$[\underline{y}', y_{mode}]$	$\uparrow$	$\uparrow$	?	?	$\leftrightarrow$	?
10	$\overline{y}' < \underline{y}' < y_{median} < \underline{y} < y_{mode} < \overline{y}$	$[\underline{y}, y_{mode}]$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\leftrightarrow$	$\uparrow$

Table 6: Effects of an increase of inequality on the preferred amount of in-kind redistribution. When the solution is a set of groups, the behavior of the most redistributive party is considered.

the set of groups able to constitute a political party. Moreover, whatever the income distribution and the diffusion of political power, political budget constraints bias the ability to participate the politics toward rich classes of population.

The second part focuses on the determination of redistributive policies in two different frameworks: the simple one considers a single policy instrument – cash redistribution – while the second investigates the joint determination of cash and in-kind redistribution. In either cases, parties commit to their genuine preferences and voters cast sincerely their ballots in favor of the party with preferences coincident to their own ones, at least for one political dimension, otherwise they abstain.

In such a framework, two cases can arise. If both inequality and political costs are low, then the policy maker may happen to be an individual more redistributive than the median voter, who implements a "high" amount of redistribution that in turns could decrease inequality and foster political participation. On the other side, if either inequality or political costs are high, then the opposite is true, and the policy maker sets an amount of redistribution lower than the preferences of the median voter. Such policies increase inequality and lead to more inequality and an even more asymmetric political power. Even if it may happen that exogenous (i.e. not due to redistributive policies) increases of inequality narrow the distance between the policy maker and the median voter if the initial inequality is very low, steadily increasing inequality leads always to a political scenario where the effective power is biased toward the rich classes of the population.

According to Director's law, preferences over cash and in-kind redistribution show different trends: while the former are always decreasing with income, the latter benefit mostly the middle class, and its utility decreases for richer and poorer individuals. The theoretical model in the paper takes this assumption into great account, generating consistent policy preferences.

The stylized facts described in the introduction, on the one side, are in line with the assumptions of the model, showing the weak correlation between ex-ante inequality and redistribution, some kinds of public expenditures and redistribution and, on the other, support the conclusions of the model that the policy makers are usually richer than the median of the population, since they set a policy mix in which cash and in-kind redistribution are positively correlated. Opposite, the two policies should be either negatively correlated – for policy makers poorer than the median – or orthogonal – for policy makers close to the middle class.

As stated in the previous sections, a much more detailed empirical investigation would be required in order to rigorously test these results, however there are some issues that make this task more difficult than what could seem: a first issue refers to the shortage of data on inequality, since comparable data are available only for the narrow subset of OECD countries.[12] Moreover, even these high-quality data disregard the effects of in-kind redistribution on the different classes of individuals that could be differently affected by it. A further argument involves the analysis of the political transmission mechanisms: even if data on electoral turnouts are provided by International Institute for Democracy and Electoral Assistance (IDEA) and other political variables and classifications are available in the World Bank Database of Political Institution (DPI), it is very difficult to investigate i) whether individuals find perfect representation in the party the vote, ii) whether, even in the most *de iure* democratic countries, there are individuals who are prevented to compete in the policy because of a shortage of resources, iii) which position in the policy space the "missing" parties would take.

[12] The Luxembourg Income Study project provides data also for some non-OECD country, but the number of observations, even if steadily increasing, is still too little.

References

AABERGE, R. AND A. LANGØRGEN, "Measuring The Benefits From Public Services: The Effects Of Local Government Spending On The Distribution Of Income In Norway," *Review of Income and Wealth* 52 (March 2006), 61–83.

ACEMOGLU, D. AND J. A. ROBINSON, "Why Did The West Extend The Franchise? Democracy, Inequality, And Growth In Historical Perspective," *The Quarterly Journal of Economics* 115 (November 2000), 1167–1199.

———, *Economic Origins Of Dictatorship And Democracy: Economic And Political Origins* (Cambridge University Press, 2005).

AGHION, P. AND P. BOLTON, "A Theory of Trickle-Down Growth and Development," *Review of Economic Studies* 64 (April 1997), 151–72.

ALESINA, A. AND D. RODRIK, "Distributive Politics and Economic Growth," *The Quarterly Journal of Economics* 109 (May 1994), 465–490.

BALDINI, M., P. BOSI AND D. PACIFICO, "Gli effetti distributivi dei trasferimenti in kind: il caso dei servizi educativi e sanitari," in A. Brandolini and C. Saraceno, eds., *Povertà e Benessere. Una geografia delle disuguaglianze in Italia* (Il Mulino, Bologna, 2007).

BERTOLA, G., "Factor Shares and Savings in Endogenous Growth," *American Economic Review* 83 (December 1993), 1184–98.

BESLEY, T. AND S. COATE, "Sources of Inefficiency in a Representative Democracy: A Dynamic Analysis," *American Economic Review* 88 (March 1998), 139–156.

BÉNABOU, R., "Inequality and Growth," NBER Working Papers 5658, National Bureau of Economic Research, Inc, May 1997.

BÉNABOU, R. AND E. A. OK, "Social Mobility And The Demand For Redistribution: The Poum Hypothesis," *The Quarterly Journal of Economics* 116 (May 2001), 447–487.

BOURGUIGNON, F. AND T. VERDIER, "Oligarchy, democracy, inequality and growth," *Journal of Development Economics* 62 (August 2000), 285–313.

D'AMBROSIO, C. AND C. GIGLIARANO, "Public health transfers in kind: measuring the distributional effects in Italy," Econpubblica Working paper series 145, Università Commerciale Luigi Bocconi, 2009.

DEININGER, K. AND L. SQUIRE, "New ways of looking at old issues: inequality and growth," *Journal of Development Economics* 57 (1998), 259–287.

GALOR, O. AND J. ZEIRA, "Income Distribution and Macroeconomics," *Review of Economic Studies* 60 (January 1993), 35–52.

GEYS, B., "Explaining voter turnout: A review of aggregate-level research," *Electoral Studies* 25 (December 2006), 637 – 663.

MARICAL, F., M. M. D'ERCOLE, M. VAALAVUO AND G. VERBIST, "Publicly-provided Services and the Distribution of Resources," OECD Social, Employment and Migration Working Papers 45, OECD, Directorate for Employment, Labour and Social Affairs, December 2006.

MELTZER, A. H. AND S. F. RICHARD, "A Rational Theory of the Size of Government," *Journal of Political Economy* 89 (October 1981), 914–27.

MILANOVIC, B., "The Median-voter Hypothesis, Income Inequality, and Income Redistribution: an Empirical Test with the Required Data," *European Journal of Political Economy* 16 (September 2000), 367–410.

MORELLI, M., "Party Formation and Policy Outcomes under Different Electoral Systems," *Review of Economic Studies* 71 (July 2004), 829–853.

PEROTTI, R., "Growth, Income Distribution, and Democracy: What the Data Say," *Journal of Economic Growth* 1 (June 1996), 149–87.

PERSSON, T. AND G. TABELLINI, "Is Inequality Harmful for Growth?," *American Economic Review* 84 (June 1994), 600–621.

SAINT-PAUL, G. AND T. VERDIER, "Education, democracy and growth," *Journal of Development Economics* 42 (December 1993), 399–407.

SCERVINI, F., "The Empirics of the Median Voter: Democracy, Redistribution and the Role of the Middle Class," LIS Working Papers 516, Luxembourg Income Studies, May 2009.

SONEDDA, D. AND G. TURATI, "Winners and Losers in the Italian Welfare State: A Microsimulation Analysis of Income Redistribution Considering In-Kind Transfers," *Giornale degli Economisti* 64 (December 2005), 423–464.

STIGLER, G. J., "Director's Law of Public Income Redistribution," *Journal of Law and Economics* 13 (April 1970), 1–10.

6 Appendices

6.1 Appendix 1 – Policy preferences

In this section I show that the functions 3 and 4 representing the individuals preferred policies satisfy the assumptions stated in section REF, and in particular, for any $y_i \in [0, y_{max}]$:

1. $\arg\max \tau(y_i) = 0$
2. $\arg\min \tau(y_i) = y_{max}$
3. $\tau'(y_i) < 0$
4. $\tau(y_i) \in [0, 1]$
5. $\arg\max \theta(y_i) = y_{median}$
6. $\arg\min \theta(y_i) = y_{max}$
7. $\theta'(y_i) > 0, \forall y_i \in [0, y_{median})$ and $\theta'(y_i) < 0, \forall y_i \in (y_{median}, y_{max}]$
8. $\theta(-y_i + y_{median}) = \theta(y_i + y_{median})$
9. $\theta(y_i) \in [0, 1]$
10. $\tau(y_i) + \theta(y_i) \in [0, 1]$

where

$$\tau(y_i) = \frac{1}{2} - \frac{y_i}{2y_{max}} \tag{62}$$

$$\theta(y_i) = \frac{1}{2} - \frac{(y_i - y_{median})^2}{2(y_{max} - y_{median})^2} \tag{63}$$

First, we observe that $\tau(y_i)$ is linear, with a negative derivative:

$$\tau'(y_i) = -\frac{1}{2y_{max}} \tag{64}$$

so that the first three assumptions are proved. Moreover, since

$$\tau(0) = \frac{1}{2}, \tau(y_{max}) = 0 \tag{65}$$

also the fourth is always satisfied.

The proofs are slightly less trivial with respect to θ. The function is a parabola, with the symmetry axes in y_{median}, indeed:

$$\theta(y_i + y_{median}) = \theta(-y_i + y_{median}) = \frac{1}{2} - \frac{y_i^2}{2(y_{max} - y_{median})^2} \tag{66}$$

As a further proof,

$$\theta'(y_i) = -\frac{(y_i - y_{median})}{(y_{max} - y_{median})^2} \begin{cases} > 0 & \text{if } y_i < y_{median} \\ = 0 & \text{if } y_i = y_{median} \\ < 0 & \text{if } y_i > y_{median} \end{cases} \tag{67}$$

and

$$\theta''(y_i) = -\frac{1}{(y_{max} - y_{median})^2} < 0 \tag{68}$$

So far, we proved assumptions 5, 7 and 8, we still need to show that $\theta(y_i)$ is minimum at y_{max}. In order to do this, given the shape and the symmetry of the function, it is enough to impose that

$$|0 - y_{median}| \le |y_{max} - y_{median}| \Rightarrow y_{max} \ge 2y_{median} \tag{69}$$

meaning that the median of the population cannot be higher than the middle point between the maximum and the minimum. As an alternative proof:

$$\theta(y_{max}) = 0 \tag{70}$$

$$\theta(0) = \frac{1}{2} - \frac{y_{median}^2}{2(y_{max} - y_{median})^2} \ge 0 \text{ if } y_{max} \ge 2y_{median} \tag{71}$$

Finally,

$$\theta(y_{median}) = \frac{1}{2}, \theta(y_{max}) = 0 \tag{72}$$

so that

$$\theta(y_i) \in \left[0, \frac{1}{2}\right] \tag{73}$$

and

$$\tau(y_i) + \theta(y_i) \in [0, 1] \tag{74}$$

6.2 Appendix 2 – Gini index

On of the possible formulations of the Gini index is the following:

$$Gini = \frac{1}{\mu(\gamma)} \int_0^{y_{max}} F(y, \gamma)(1 - F(y, \gamma))\, dy \tag{75}$$

where $\mu(\gamma)$ is the mean of the population and $F(y, \gamma)$ is the cumulative distribution function.

We need to prove that the derivative of the Gini index with respect to the parameter γ is positive. By assumption we know that the mean decreases with γ while, by construction, we know that $F(y, \gamma)$ increases with it.

The derivative with respect to γ is:

$$\frac{\partial Gini}{\partial \gamma} = \tag{76}$$

$$= -\frac{\mu(\gamma)'}{\mu(\gamma)^2} \int_0^{y_{max}} F(y,\gamma)(1-F(y,\gamma))\,dy + \tag{77}$$

$$+\frac{1}{\mu(\gamma)} \frac{\partial \int_0^{y_{max}} F(y,\gamma)(1-F(y,\gamma))\,dy}{\partial \gamma} = \tag{78}$$

$$= \frac{1}{\mu(\gamma)} \left(\frac{\partial \int_0^{y_{max}} F(y,\gamma)(1-F(y,\gamma))\,dy}{\partial \gamma} - \mu(\gamma)' Gini \right) > 0 \tag{79}$$

$$\text{if } \frac{\partial \int_0^{y_{max}} F(y,\gamma)(1-F(y,\gamma))\,dy}{\partial \gamma} > \mu(\gamma)' Gini \tag{80}$$

$$\Rightarrow \frac{\partial \int_0^{y_{max}} F(y,\gamma)(1-F(y,\gamma))\,dy}{\partial \gamma} > 0 \tag{81}$$

$$\text{since } \frac{1}{\mu(\gamma)} > 0, \mu(\gamma)' < 0, Gini > 0 \tag{82}$$

By construction, we know that an increment of γ shifts upwards the function $F(y,\gamma)$, so that the integral over the whole support is positive, that is

$$\frac{\partial \int_0^{y_{max}} F(y,\gamma)\,dy}{\partial \gamma} > 0 \tag{83}$$

6.3 Appendix 3 – Winner party in the two-policy space

First of all, it is needed to set some notation and to identify how many possible cases should be analyzed. Call a generic income y_i and its symmetric with respect to the median $y_i' = 2y_{median} - y_i$. Analogously, there are $\overline{y}'$ as the symmetric of $\overline{y}$ and $\underline{y}'$ as the symmetric of $\underline{y}'$. By construction, we know that $\overline{y}' < y_{median} < \overline{y}$. Also, by construction we know that either $\underline{y} < y_{median} < \underline{y}'$ or $\underline{y}' < y_{median} < \underline{y}$. Moreover, by definition $\underline{y} < \overline{y}$ and, therefore, $\overline{y}' < \underline{y}'$. There are, finally, ten possible cases:

1. $y_{mode} < \overline{y}' < \underline{y} < y_{median} < \underline{y}' < \overline{y}$
2. $y_{mode} < \overline{y}' < \underline{y}' < y_{median} < \underline{y} < \overline{y}$
3. $\overline{y}' < y_{mode} < \underline{y} < y_{median} < \underline{y}' < \overline{y}$
4. $\overline{y}' < y_{mode} < \underline{y}' < y_{median} < \underline{y} < \overline{y}$
5. $\overline{y}' < \underline{y} < y_{mode} < y_{median} < \underline{y}' < \overline{y}$
6. $\overline{y}' < \underline{y}' < y_{mode} < y_{median} < \underline{y} < \overline{y}$
7. $\overline{y}' < \underline{y} < y_{median} < y_{mode} < \underline{y}' < \overline{y}$

8. $\overline{y}' < \underline{y}' < y_{median} < y_{mode} < \underline{y} < \overline{y}$
9. $\overline{y}' < \underline{y} < y_{median} < \underline{y}' < y_{mode} < \overline{y}$
10. $\overline{y}' < \underline{y}' < y_{median} < \underline{y} < y_{mode} < \overline{y}$

6.3.1 Case 1

$$y_{mode} < \overline{y}' < \underline{y} < y_{median} < \underline{y}' < \overline{y} \tag{84}$$

$$vote\left(y_i\right) = \begin{cases} y_i & \text{if } y_i \in \left[\underline{y}, \overline{y}\right] \\ y_i' \in \left[\underline{y}', \overline{y}\right] & \text{if } y_i \in \left[\overline{y}', \underline{y}\right] \\ \emptyset & \text{if } y_i \in [0, \overline{y}'] \cup [\overline{y}, y_{max}] \end{cases} \tag{85}$$

$$\text{If } y_i \in \left[\underline{y}, \underline{y}'\right] \Rightarrow v\left(y_i\right) = f\left(y_i\right) \tag{86}$$
$$f\left(\underline{y}\right) > f\left(y_i\right) \Rightarrow \arg\max v\left(y_i\right) = \underline{y} \tag{87}$$
$$\text{since } f\left(\underline{y}\right) > f\left(y_i\right), \forall y_i \in \left[\underline{y}, \underline{y}'\right] \tag{88}$$

$$\text{If } y_i \in \left[\underline{y}', \overline{y}\right] \Rightarrow v\left(y_i\right) = f\left(y_i\right) + f\left(y_i'\right) \tag{89}$$
$$\Rightarrow \arg\max v\left(y_i\right) \in \left[\underline{y}', \overline{y}\right] \tag{90}$$

since nothing can be said about the winning party.

$$w_i = y_i \in \left[\underline{y}', \overline{y}\right] \tag{91}$$
$$\text{since } f\left(\underline{y}\right) < f\left(y_i\right), \forall y_i \in \left[\underline{y}', \overline{y}\right] \tag{92}$$

6.3.2 Case 2

$$y_{mode} < \overline{y}' < \underline{y}' < y_{median} < \underline{y} < \overline{y} \tag{93}$$

$$vote\left(y_i\right) = \begin{cases} y_i & \text{if } y_i \in \left[\underline{y}, \overline{y}\right] \\ y_i' \in \left[\underline{y}, \overline{y}\right] & \text{if } y_i \in \left[\overline{y}', \underline{y}'\right] \\ \emptyset & \text{if } y_i \in [0, \overline{y}'] \cup \left[\underline{y}', \underline{y}\right] \cup [\overline{y}, y_{max}] \end{cases} \tag{94}$$

$$\text{If } y_i \in \left[\underline{y}, \overline{y}\right] \Rightarrow v\left(y_i\right) = f\left(y_i\right) + f\left(y_i'\right) \tag{95}$$
$$\Rightarrow \arg\max v\left(y_i\right) \in \left[\underline{y}, \overline{y}\right] \tag{96}$$

since nothing can be said about the winning party.

$$w_i = y_i \in \left[\underline{y}, \overline{y}\right] \tag{97}$$

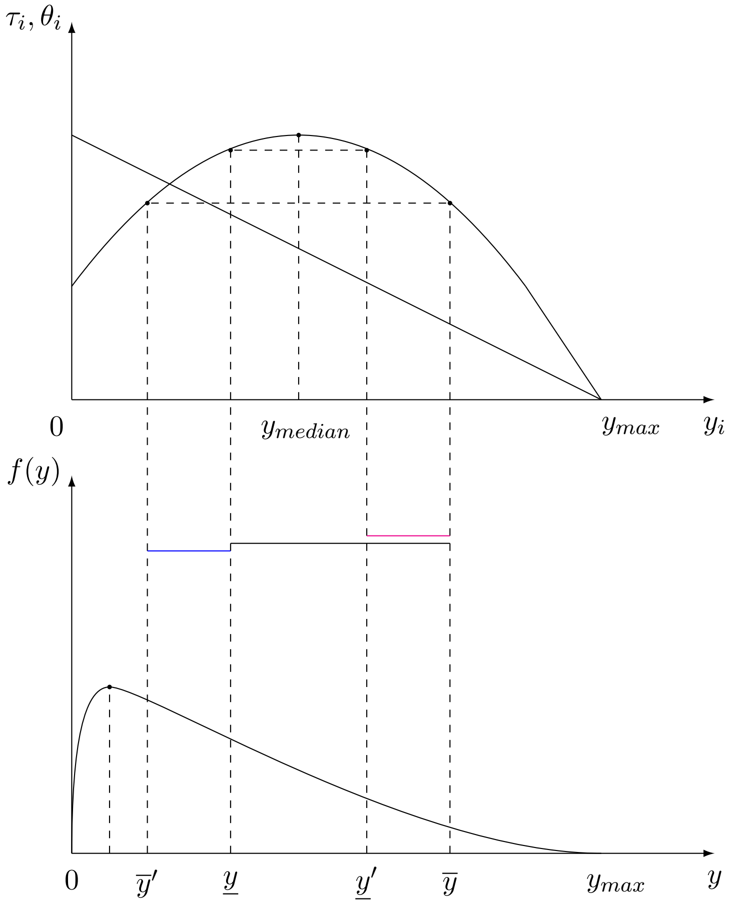


Figure 8: Case 1

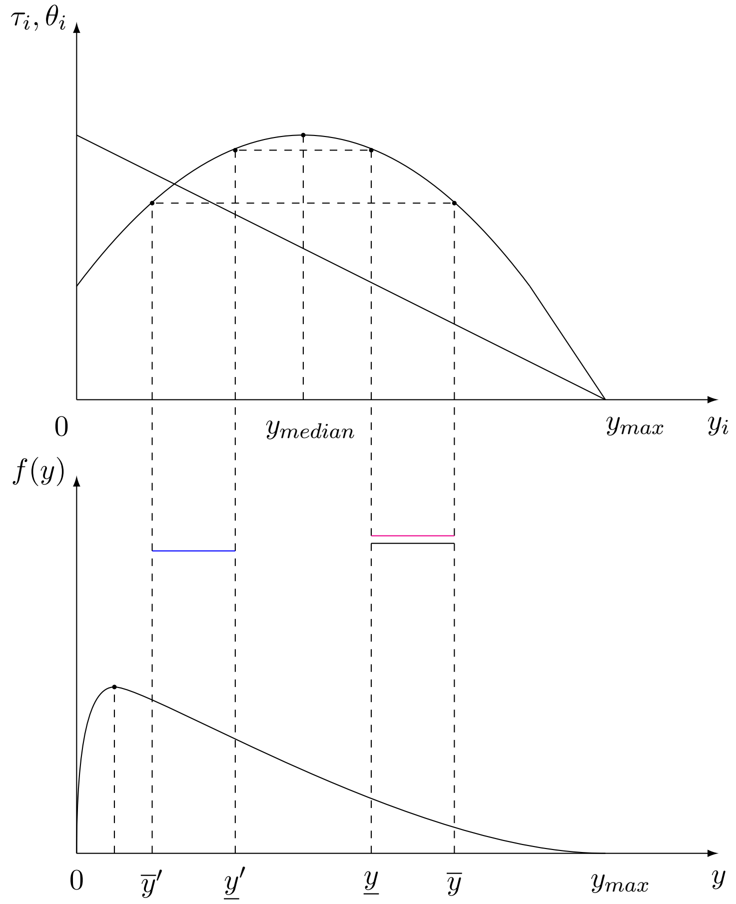


Figure 9: Case 2

6.3.3 Case 3

$$\overline{y}' < y_{mode} < \underline{y} < y_{median} < \underline{y}' < \overline{y} \tag{98}$$

$$vote\left(y_i\right) = \begin{cases} y_i & \text{if } y_i \in \left[\underline{y}, \overline{y}\right] \\ y_i' \in \left[\underline{y}', \overline{y}\right] & \text{if } y_i \in \left[\overline{y}', \underline{y}\right] \\ \emptyset & \text{if } y_i \in [0, \overline{y}'] \cup [\overline{y}, y_{max}] \end{cases} \tag{99}$$

$$\text{If } y_i \in \left[\underline{y}, \underline{y}'\right] \Rightarrow v\left(y_i\right) = f\left(y_i\right) \tag{100}$$

$$f\left(\underline{y}\right) > f\left(y_i\right) \Rightarrow \arg\max v\left(y_i\right) = \underline{y} \tag{101}$$

$$\text{since } f\left(\underline{y}\right) > f\left(y_i\right), \forall y_i \in \left[\underline{y}, \underline{y}'\right] \tag{102}$$

$$\text{If } y_i \in \left[\underline{y}', y'_{mode}\right] \Rightarrow v\left(y_i\right) = f\left(y_i\right) + f\left(y_i'\right) \tag{103}$$

$$f\left(y'_{mode}\right) + f\left(y_{mode}\right) ? f\left(y_i\right) + f\left(y_i'\right) \Rightarrow \arg\max v\left(y_i\right) \in \left[\underline{y}', y'_{mode}\right] \tag{104}$$

$$\text{since } f\left(y_{mode}\right) > f\left(y_i'\right) \text{ but } f\left(y'_{mode}\right) < f\left(y_i\right), \forall y_i \in \left[\underline{y}', y'_{mode}\right] \tag{105}$$

$$\text{If } y_i \in \left[y'_{mode}, \overline{y}\right] \Rightarrow v\left(y_i\right) = f\left(y_i\right) + f\left(y_i'\right) \tag{106}$$

$$f\left(y'_{mode}\right) + f\left(y_{mode}\right) > f\left(y_i\right) + f\left(y_i'\right) \Rightarrow \arg\max v\left(y_i\right) = y'_{mode} \tag{107}$$

$$\text{since } f\left(y_{mode}\right) > f\left(y_i'\right) \text{ and } f\left(y'_{mode}\right) > f\left(y_i\right), \forall y_i \in \left[y'_{mode}, \overline{y}\right] \tag{108}$$

$$w_i = y_i \in \left[\underline{y}', y'_{mode}\right] \tag{109}$$

$$\text{since } f\left(\underline{y}\right) < f\left(y_i\right), \forall y_i \in \left[y'_{mode}, \overline{y}\right] \tag{110}$$

6.3.4 Case 4

$$\overline{y}' < y_{mode} < \underline{y}' < y_{median} < \underline{y} < \overline{y} \tag{111}$$

$$vote\left(y_i\right) = \begin{cases} y_i & \text{if } y_i \in \left[\underline{y}, \overline{y}\right] \\ y_i' \in \left[\underline{y}, \overline{y}\right] & \text{if } y_i \in \left[\overline{y}', \underline{y}'\right] \\ \emptyset & \text{if } y_i \in [0, \overline{y}'] \cup \left[\underline{y}', \underline{y}\right] \cup [\overline{y}, y_{max}] \end{cases} \tag{112}$$

$$\text{If } y_i \in \left[\underline{y}, y'_{mode}\right] \Rightarrow v\left(y_i\right) = f\left(y_i\right) + f\left(y_i'\right) \tag{113}$$

$$f\left(y'_{mode}\right) + f\left(y_{mode}\right) ? f\left(y_i\right) + f\left(y_i'\right) \Rightarrow \arg\max v\left(y_i\right) \in \left[\underline{y}, y'_{mode}\right] \tag{114}$$

$$\text{since } f\left(y_{mode}\right) > f\left(y_i'\right) \text{ but } f\left(y'_{mode}\right) < f\left(y_i\right), \forall y_i \in \left[\underline{y}, y'_{mode}\right] \tag{115}$$

$$\text{If } y_i \in \left[y'_{mode}, \overline{y}\right] \Rightarrow v\left(y_i\right) = f\left(y_i\right) + f\left(y_i'\right) \tag{116}$$

$$f\left(y'_{mode}\right) + f\left(y_{mode}\right) > f\left(y_i\right) + f\left(y_i'\right) \Rightarrow \arg\max v\left(y_i\right) = y'_{mode} \tag{117}$$

$$\text{since } f\left(y_{mode}\right) > f\left(y_i'\right) \text{ and } f\left(y'_{mode}\right) > f\left(y_i\right), \forall y_i \in \left[y'_{mode}, \overline{y}\right] \tag{118}$$

$$w_i = y_i \in \left[\underline{y}, y'_{mode}\right] \tag{119}$$

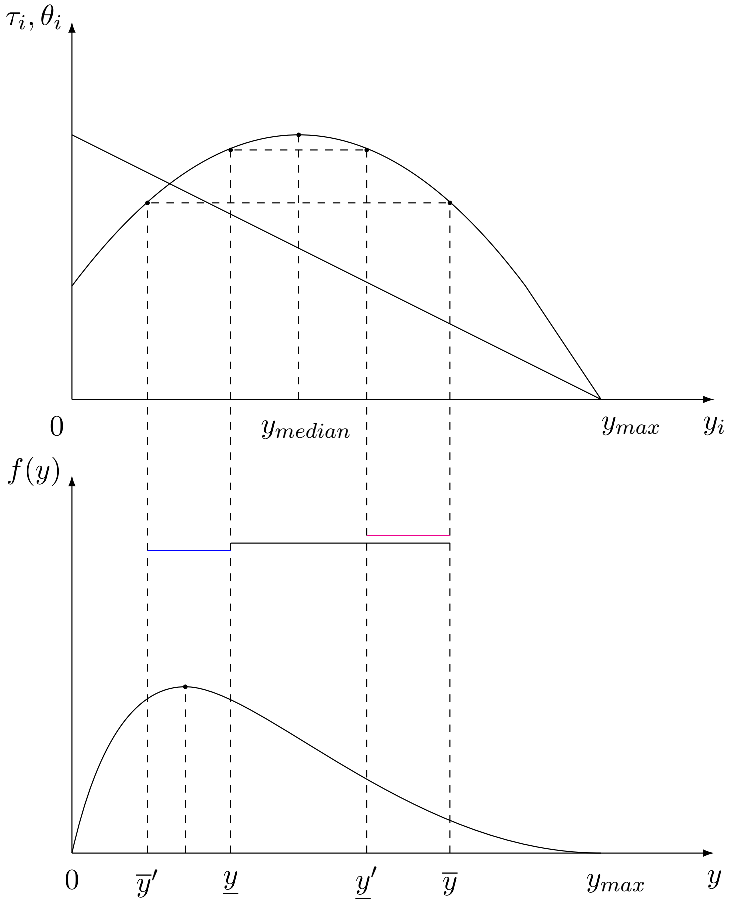


Figure 10: Case 3

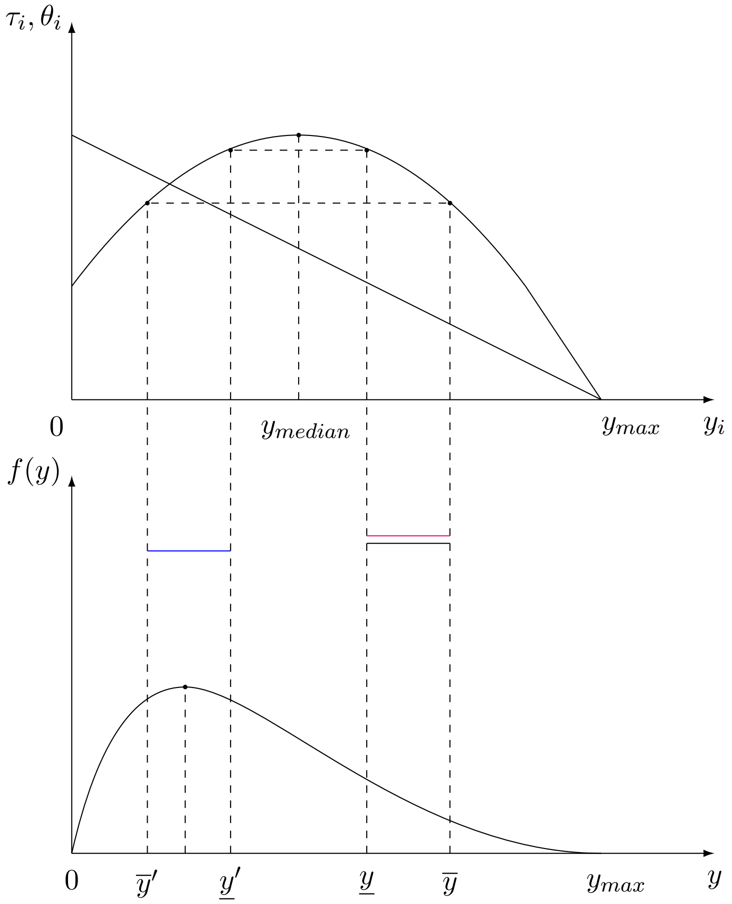


Figure 11: Case 4

6.3.5 Cases 5 and 7

$$\overline{y}' < \underline{y} < y_{mode} < y_{median} < \underline{y}' < \overline{y} \tag{120}$$
$$\overline{y}' < \underline{y} < y_{median} < y_{mode} < \underline{y}' < \overline{y} \tag{121}$$

$$vote\,(y_i) = \begin{cases} y_i & \text{if } y_i \in \left[\underline{y}, \overline{y}\right] \\ y_i' \in \left[\underline{y}', \overline{y}\right] & \text{if } y_i \in \left[\overline{y}', \underline{y}\right] \\ \emptyset & \text{if } y_i \in [0, \overline{y}'] \cup [\overline{y}, y_{max}] \end{cases} \tag{122}$$

$$\text{If } y_i \in \left[\underline{y}, \underline{y}'\right] \Rightarrow v\,(y_i) = f\,(y_i) \tag{123}$$
$$\arg\max v\,(y_i) = y_{mode} \text{ by definition of mode.} \tag{124}$$

$$\text{If } y_i \in \left[\underline{y}', \overline{y}\right] \Rightarrow v\,(y_i) = f\,(y_i) + f\left(y_i'\right) \tag{125}$$
$$f\left(\underline{y}'\right) + f\left(\underline{y}\right) > f\,(y_i) + f\,(y_i) \Rightarrow \arg\max v\,(y_i) = \underline{y}' \tag{126}$$
$$\text{since } f\left(\underline{y}'\right) > f\,(y_i) \text{ and } f\left(\underline{y}\right) > f\left(y_i'\right), \forall y_i \in \left[\underline{y}', \overline{y}\right] \tag{127}$$

$$w_i = \begin{cases} y_{mode} & \text{if } f\,(y_{mode}) > f\left(\underline{y}'\right) + f\left(\underline{y}\right) \\ \underline{y}' & \text{if } f\,(y_{mode}) < f\left(\underline{y}'\right) + f\left(\underline{y}\right) \end{cases} \tag{128}$$

6.3.6 Cases 6 and 8

$$\overline{y}' < \underline{y}' < y_{mode} < y_{median} < \underline{y} < \overline{y} \tag{129}$$
$$\overline{y}' < \underline{y}' < y_{median} < y_{mode} < \underline{y} < \overline{y} \tag{130}$$

$$vote\,(y_i) = \begin{cases} y_i & \text{if } y_i \in \left[\underline{y}, \overline{y}\right] \\ y_i' \in \left[\underline{y}, \overline{y}\right] & \text{if } y_i \in \left[\overline{y}', \underline{y}'\right] \\ \emptyset & \text{if } y_i \in [0, \overline{y}'] \cup \left[\underline{y}', \underline{y}\right] \cup [\overline{y}, y_{max}] \end{cases} \tag{131}$$

$$\text{If } y_i \in \left[\underline{y}, \overline{y}\right] \Rightarrow v\,(y_i) = f\,(y_i) + f\left(y_i'\right) \tag{132}$$
$$f\left(\underline{y}\right) + f\left(\underline{y}'\right) > f\,(y_i) + f\left(y_i'\right) \Rightarrow \arg\max v\,(y_i) = \underline{y} \tag{133}$$
$$\text{since } f\left(\underline{y}\right) > f\,(y_i) \text{ and } f\left(\underline{y}'\right) > f\,(y_i), \forall y_i \in \left[\underline{y}, \overline{y}\right] \tag{134}$$

$$w_i = \underline{y} \tag{135}$$

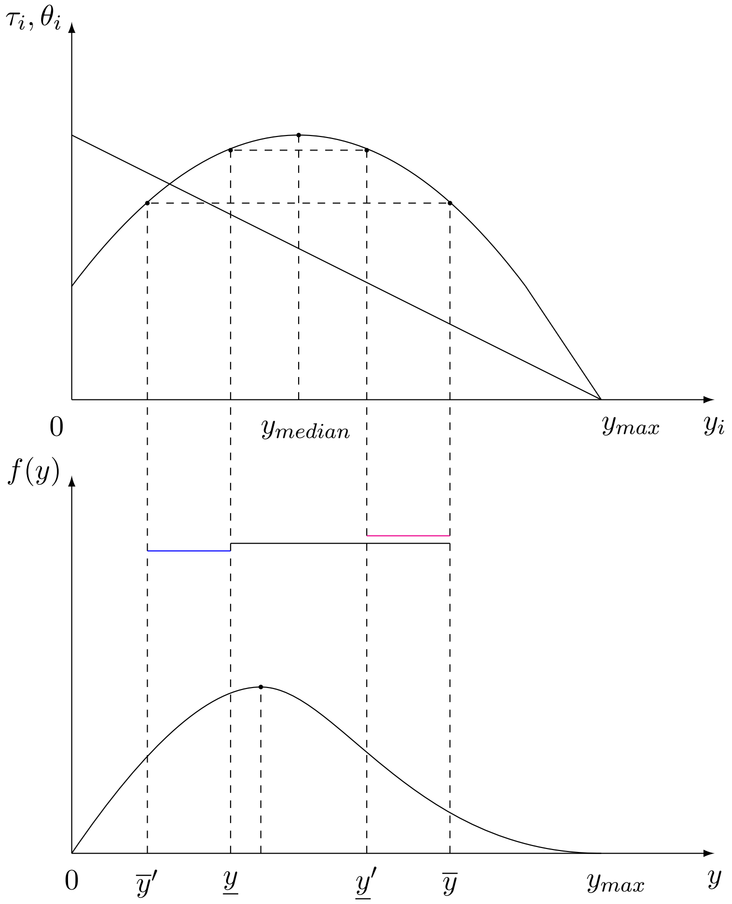


Figure 12: Cases 5 and 7

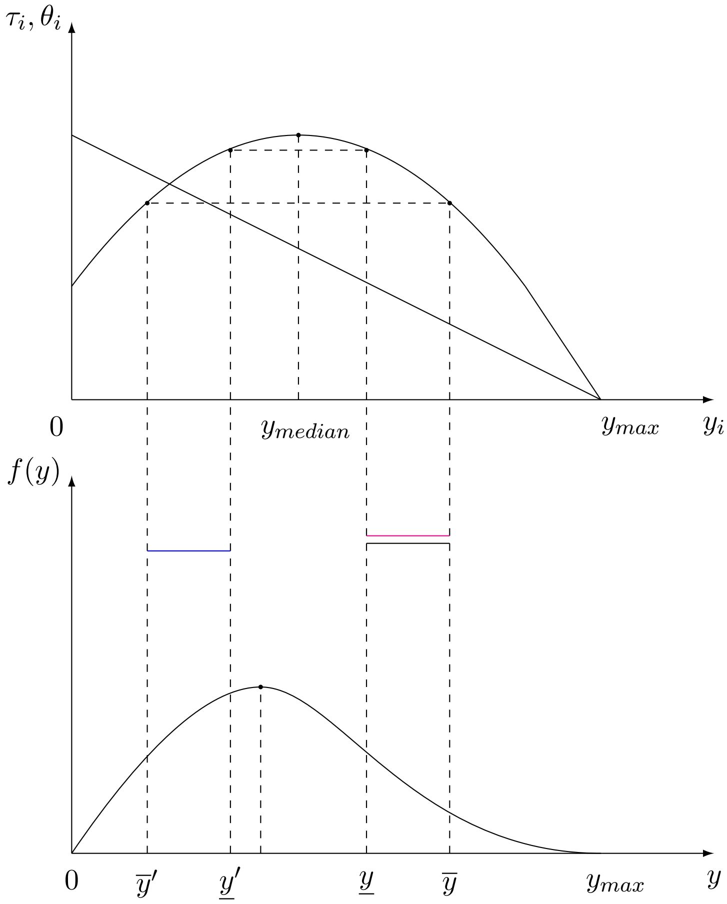


Figure 13: Cases 6 and 8

6.3.7 Case 9

$$\overline{y}' < \underline{y} < y_{median} < \underline{y}' < y_{mode} < \overline{y} \tag{136}$$

$$vote\left(y_i\right) = \begin{cases} y_i & \text{if } y_i \in \left[\underline{y}, \overline{y}\right] \\ y_i' \in \left[\underline{y}', \overline{y}\right] & \text{if } y_i \in \left[\overline{y}', \underline{y}\right] \\ \emptyset & \text{if } y_i \in [0, \overline{y}'] \cup [\overline{y}, y_{max}] \end{cases} \tag{137}$$

$$\text{If } y_i \in \left[\underline{y}, \underline{y}'\right] \Rightarrow v\left(y_i\right) = f\left(y_i\right) \tag{138}$$

$$f\left(\underline{y}'\right) > f\left(y_i\right) \Rightarrow \arg\max v\left(y_i\right) = \underline{y}' \tag{139}$$

$$\text{since } f\left(\underline{y}'\right) > f\left(y_i\right), \forall y_i \in \left[\underline{y}, \underline{y}'\right] \tag{140}$$

$$\text{If } y_i \in \left[\underline{y}', y_{mode}\right] \Rightarrow v\left(y_i\right) = f\left(y_i\right) + f\left(y_i'\right) \tag{141}$$

$$f\left(y_{mode}\right) + f\left(y_{mode}'\right) ? f\left(y_i\right) + f\left(y_i'\right) \Rightarrow \arg\max v\left(y_i\right) \in \left[\underline{y}', y_{mode}\right] \tag{142}$$

$$\text{since } f\left(y_{mode}\right) > f\left(y_i\right) \text{ but } f\left(y_{mode}'\right) < f\left(y_i'\right), \forall y_i \in \left[\underline{y}', y_{mode}\right] \tag{143}$$

$$\text{If } y_i \in \left[y_{mode}, \overline{y}\right] \Rightarrow v\left(y_i\right) = f\left(y_i\right) + f\left(y_i'\right) \tag{144}$$

$$f\left(y_{mode}\right) + f\left(y_{mode}'\right) > f\left(y_i\right) + f\left(y_i'\right) \Rightarrow \arg\max v\left(y_i\right) = y_{mode} \tag{145}$$

$$\text{since } f\left(y_{mode}\right) > f\left(y_i\right) \text{ and } f\left(y_{mode}'\right) > f\left(y_i'\right), \forall y_i \in \left[y_{mode}, \overline{y}\right] \tag{146}$$

$$w_i = y_i \in \left[\underline{y}', y_{mode}\right] \tag{147}$$

$$\text{since } f\left(\underline{y}'\right) < f\left(y_i\right), \forall y_i \in \left(\underline{y}', y_{mode}\right] \tag{148}$$

6.3.8 Case 10

$$\overline{y}' < \underline{y}' < y_{median} < \underline{y} < y_{mode} < \overline{y} \tag{149}$$

$$vote\left(y_i\right) = \begin{cases} y_i & \text{if } y_i \in \left[\underline{y}, \overline{y}\right] \\ y_i' \in \left[\underline{y}, \overline{y}\right] & \text{if } y_i \in \left[\overline{y}', \underline{y}'\right] \\ \emptyset & \text{if } y_i \in [0, \overline{y}'] \cup \left[\underline{y}', \underline{y}\right] \cup [\overline{y}, y_{max}] \end{cases} \tag{150}$$

$$\text{If } y_i \in \left[y_{mode}, \overline{y}\right] \Rightarrow v\left(y_i\right) = f\left(y_i\right) + f\left(y_i'\right) \tag{151}$$

$$f\left(y_{mode}\right) + f\left(y_{mode}'\right) > f\left(y_i\right) + f\left(y_i\right) \Rightarrow \arg\max v\left(y_i\right) = y_{mode} \tag{152}$$

$$\text{since } f\left(y_{mode}\right) > f\left(y_i\right) \text{ and } f\left(y_{mode}'\right) > f\left(y_i'\right), \forall y_i \in \left[y_{mode}, \overline{y}\right] \tag{153}$$

$$\text{If } y_i \in \left[\underline{y}, y_{mode}\right] \Rightarrow v\left(y_i\right) = f\left(y_i\right) + f\left(y_i'\right) \tag{154}$$

$$f\left(y_{mode}\right) + f\left(y_{mode}'\right) ? f\left(y_i\right) + f\left(y_i\right) \Rightarrow \arg\max v\left(y_i\right) \in \left[\underline{y}, y_{mode}\right] \tag{155}$$

$$\text{since } f\left(y_{mode}\right) > f\left(y_i\right) \text{ but } f\left(y_{mode}'\right) < f\left(y_i'\right), \forall y_i \in \left[\underline{y}, y_{mode}\right] \tag{156}$$

$$w_i = y_i \in \left[\underline{y}, y_{mode}\right] \tag{157}$$

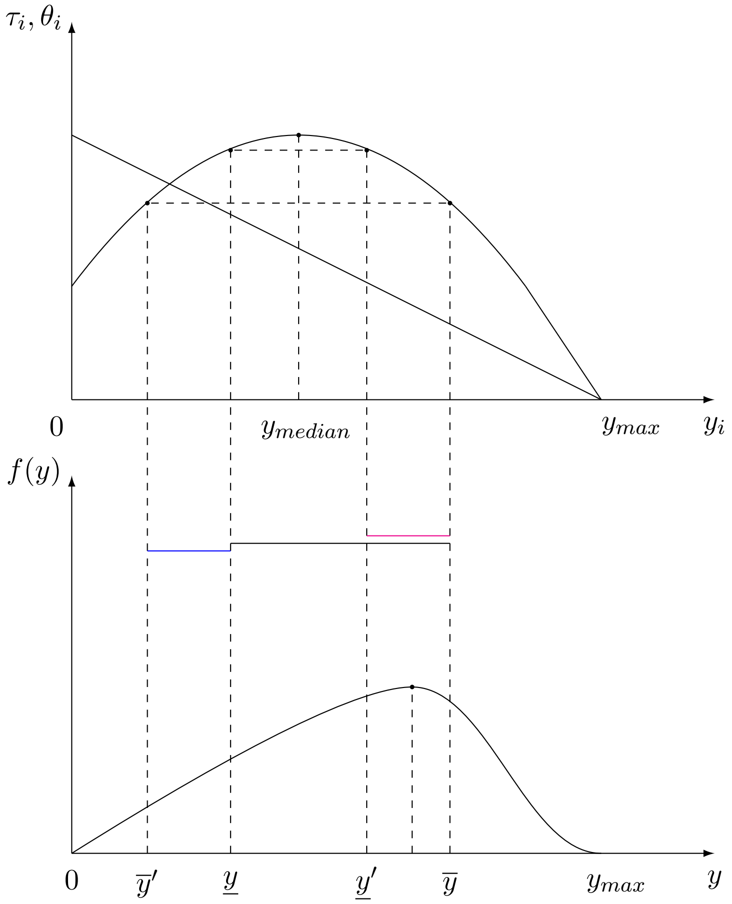


Figure 14: Case 9

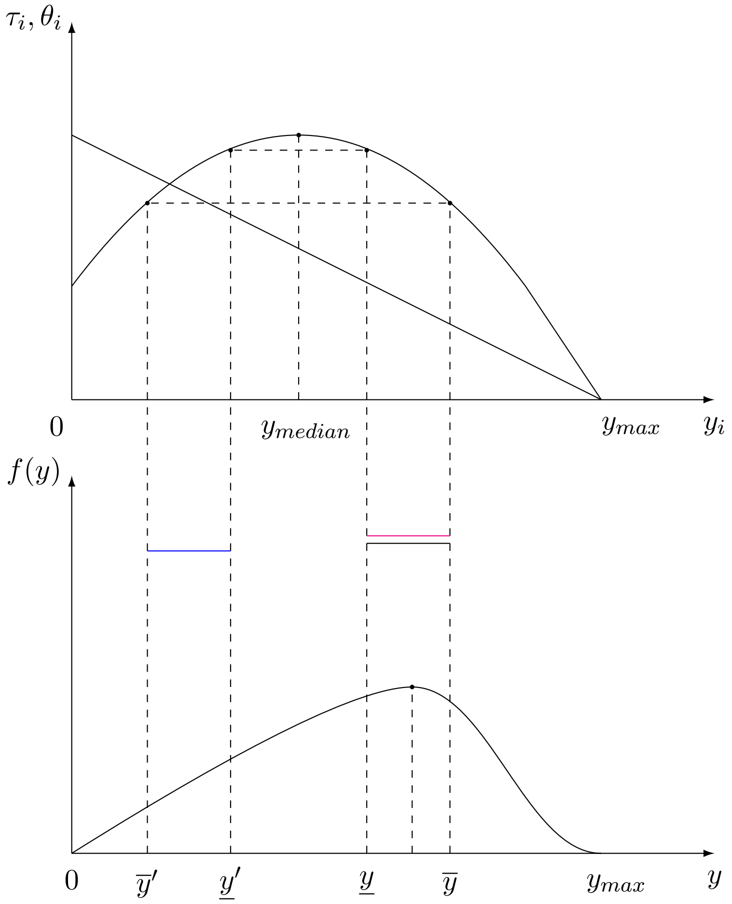


Figure 15: Case 10

DEPARTMENT OF ECONOMICS AND PUBLIC FINANCE “G. PRATO”
UNIVERSITY OF TORINO
Corso Unione Sovietica 218 bis - 10134 Torino (ITALY)
Phone: +39 011 6706128 - Fax: +39 011 6706062
Web page: http://eco83.econ.unito.it/prato/